PenCon2018
Łódź, Poland

AECS & IMA2018
Narita, Japan

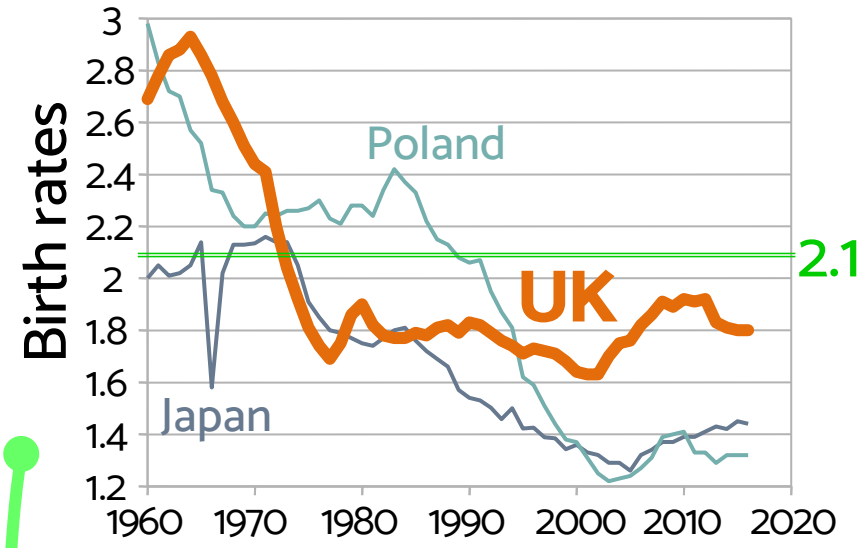
Forecasting the impact of **state pension reforms** in **post-Brexit England & Wales** using **microsimulation** and **deep learning**

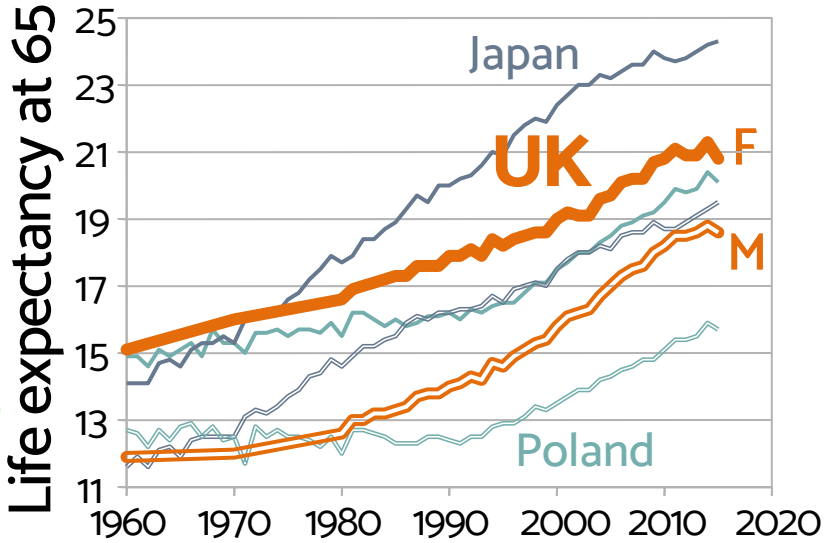
Agnieszka Werpachowska

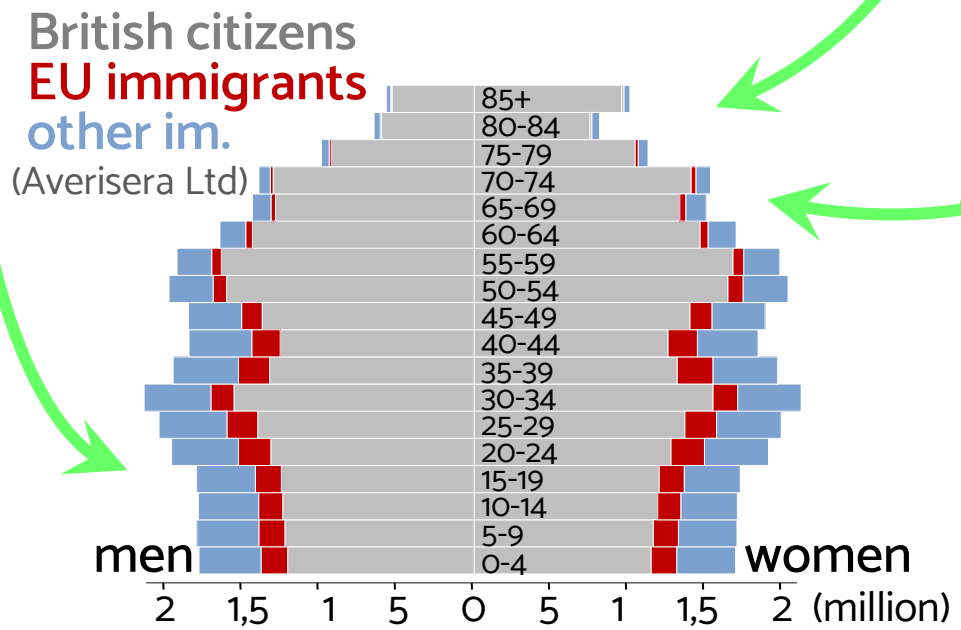
AVERISERA
MATHEMATICAL INVESTIGATIONS OF REALITY

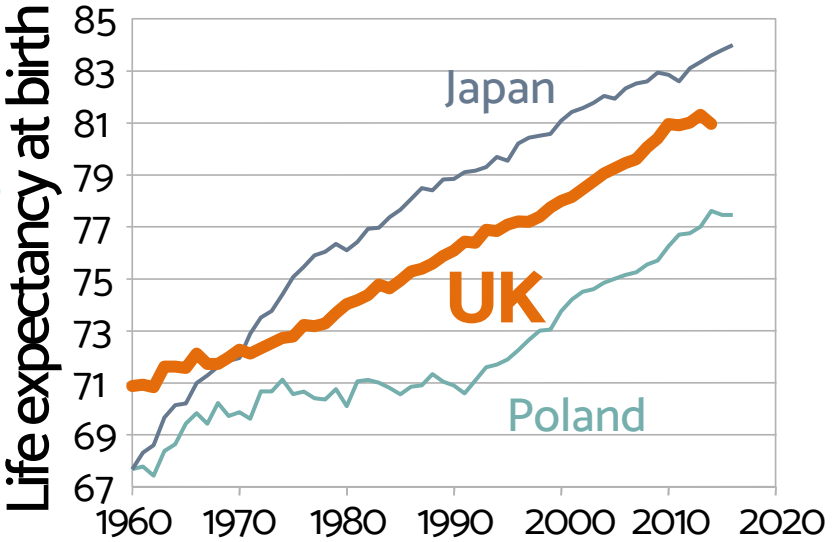
averisera.uk

Population ageing in advanced economies



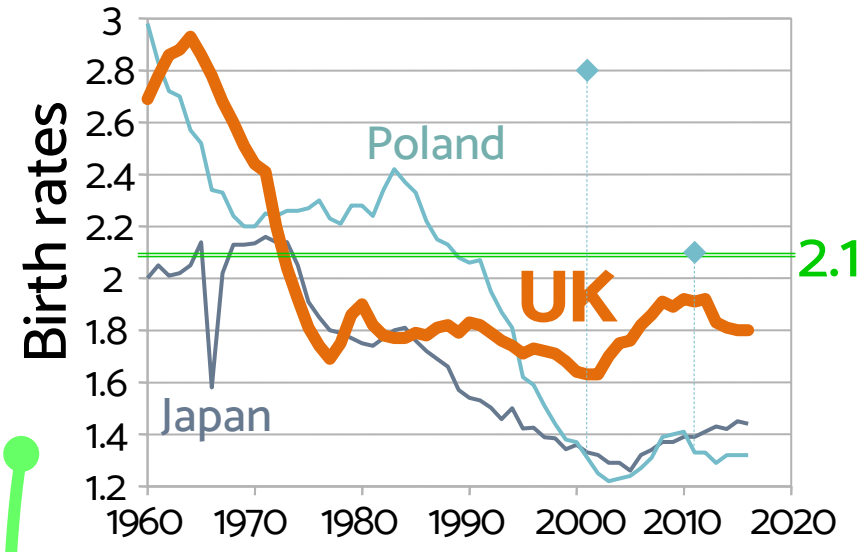


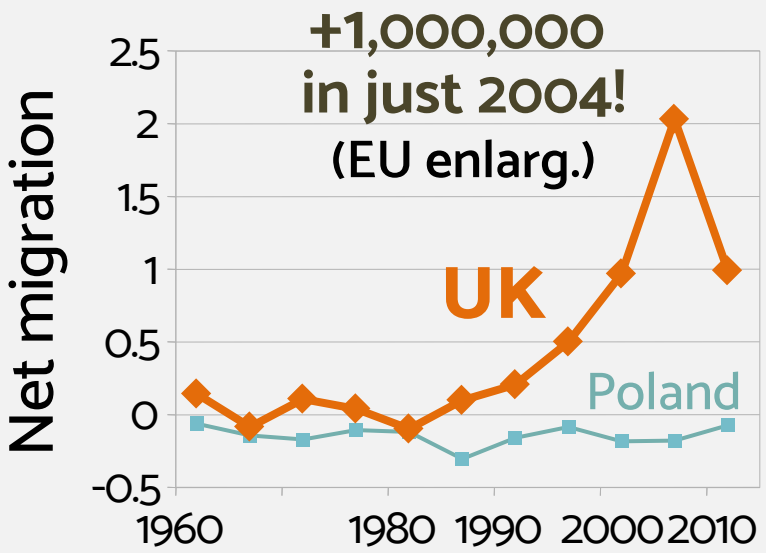


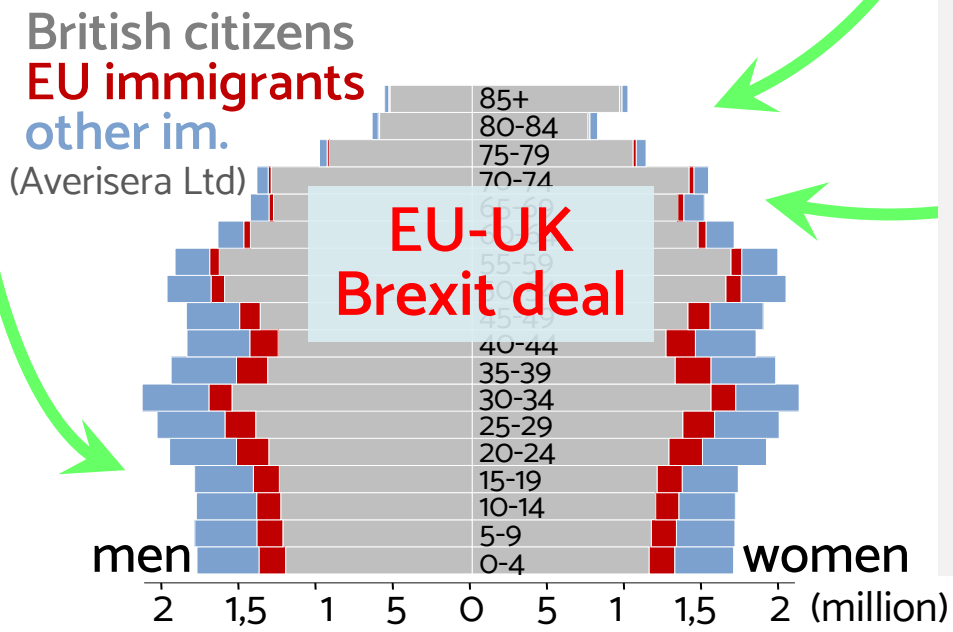


(OECD, WB)

Population ageing in advanced economies





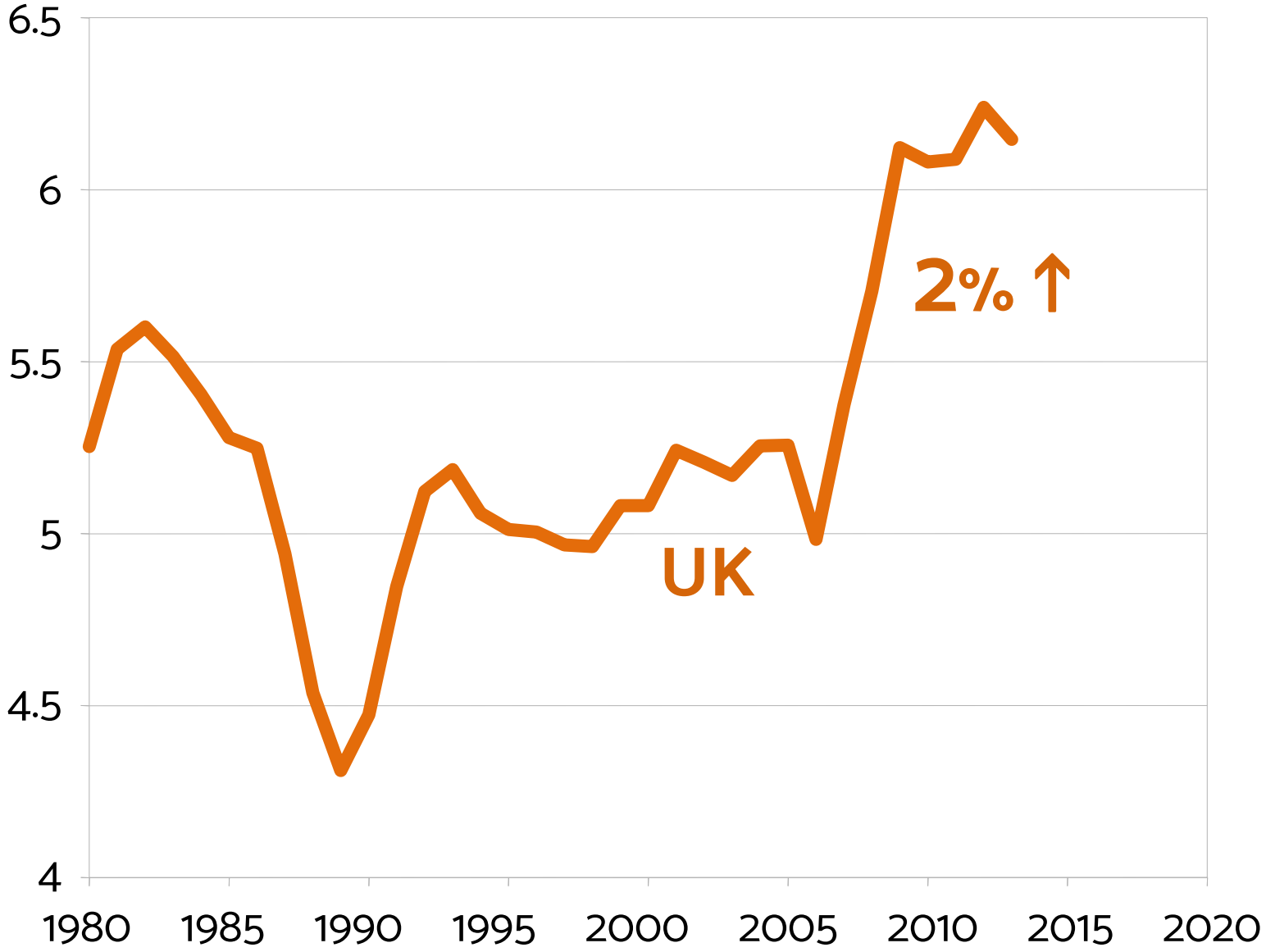


"Microsimulations of demographic changes in England & Wales under different EU referendum scenarios", *Int. J. of Microsimulation*, 10(2), 2017:

specific age & sex structure of EU immigration: young & more women than the UK population average

(OECD, WB)

State pension cost (% of GDP)



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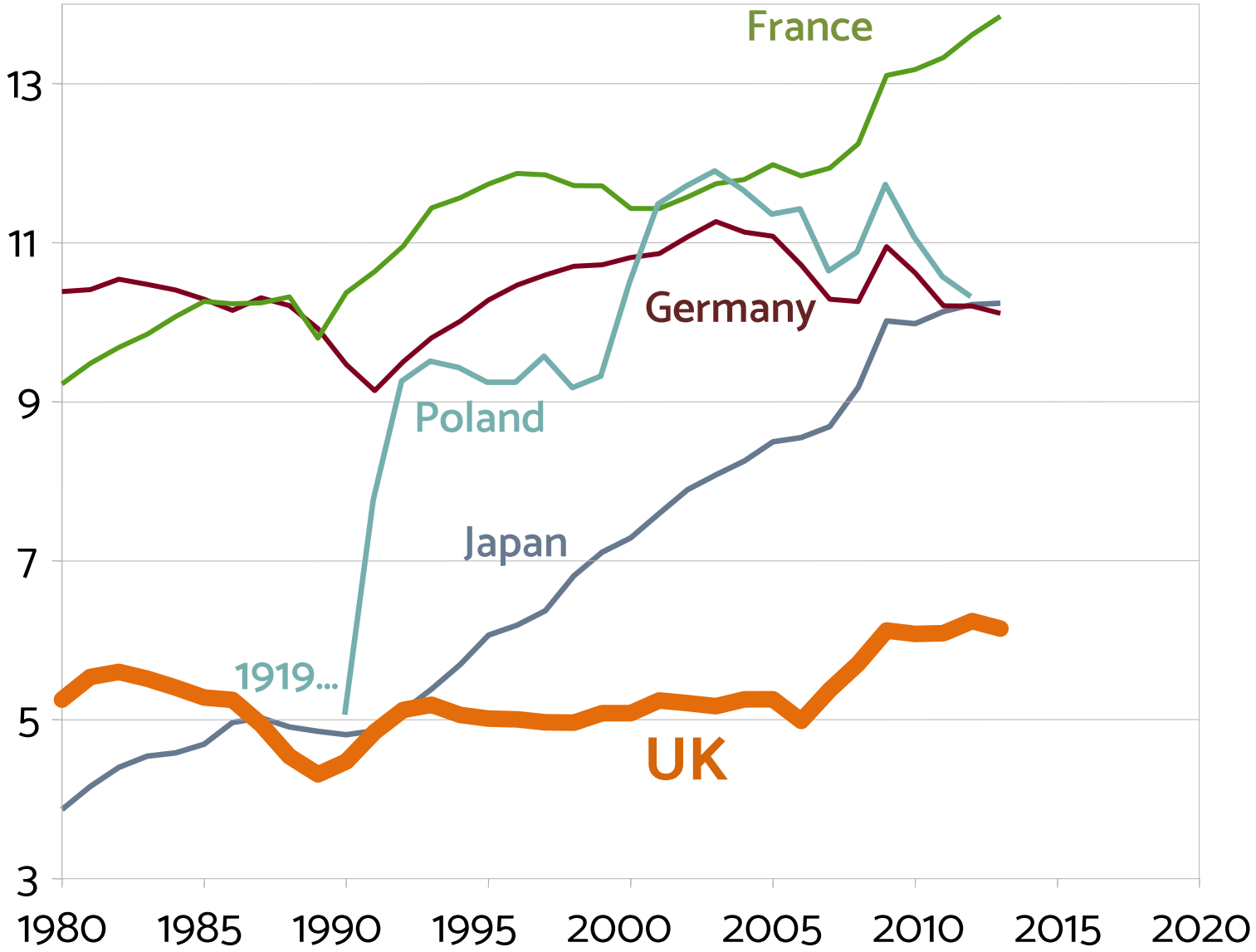

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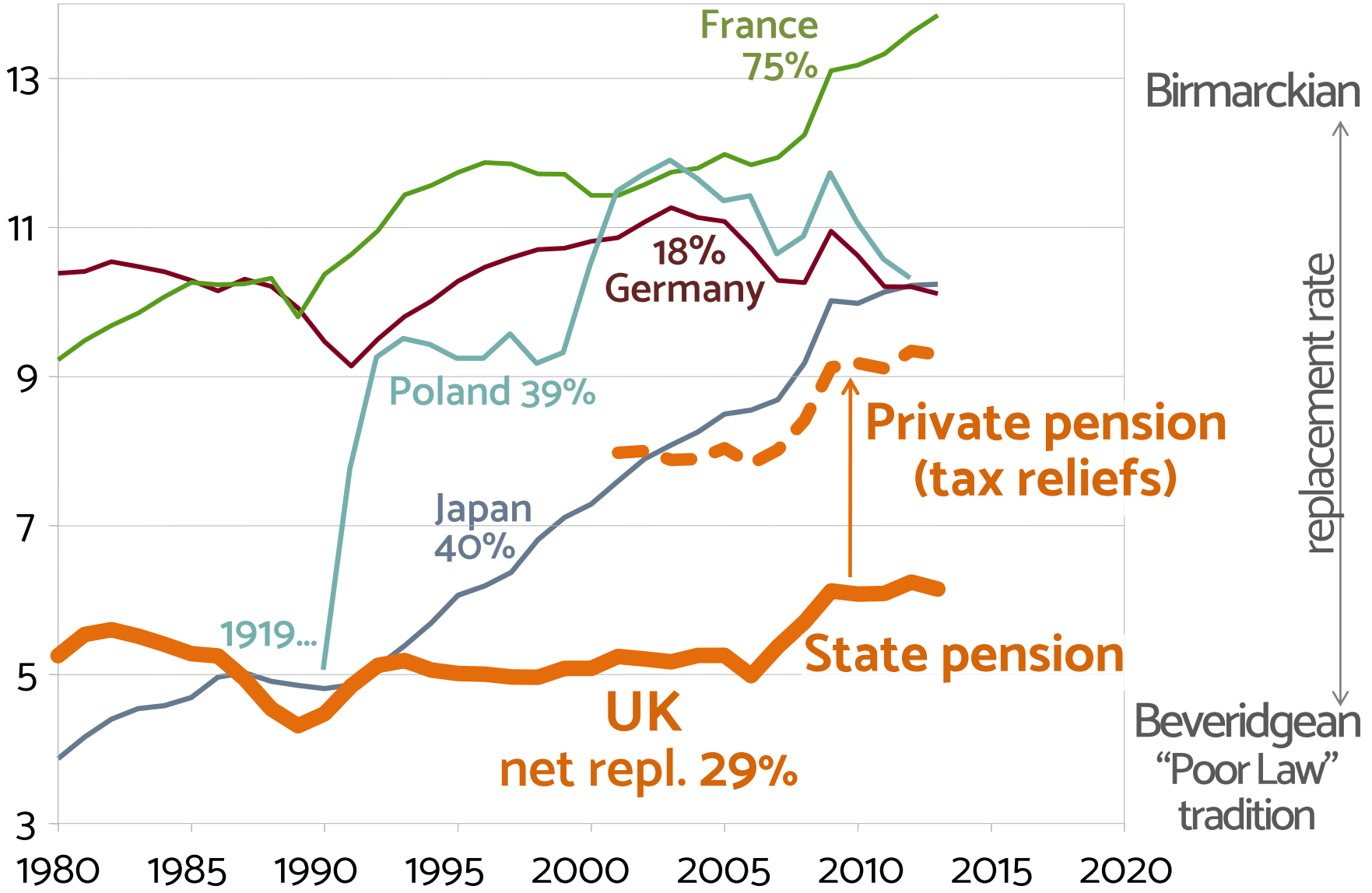


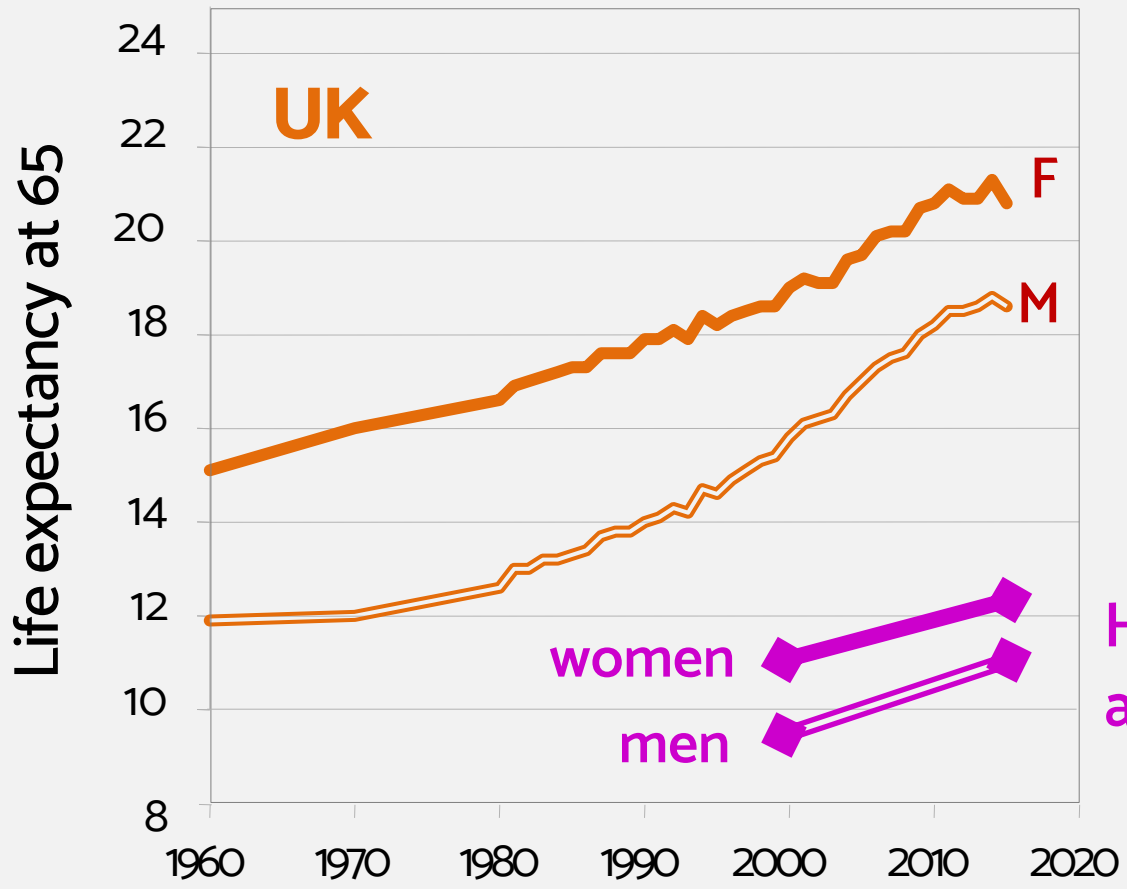
Population ageing: effects & mitigation

- Shrinking workforce (relative to retirees)
  - **Encourage immigration** (politically controversial & social expenditure)
  - **Raise the retirement age**

- Growing pensions expenditure
  - **State pension cuts** (politically unpopular; "triple lock", ≥ 2.5% p.a.)
  - **Spending cuts and tax rises across the board & austerity** (falling living standards, workers' incentive and economic growth)
  - **Borrowing** (national debt)
  - **Raise the retirement age**

- Growing spending on other social benefits and healthcare

Population ageing: effects & mitigation



ONS, UK

HALE at 65 (health-adjusted life exp.)

- Growing spending on other social benefits and healthcare
- Raise the retirement age by a few years

Population ageing: effects & mitigation

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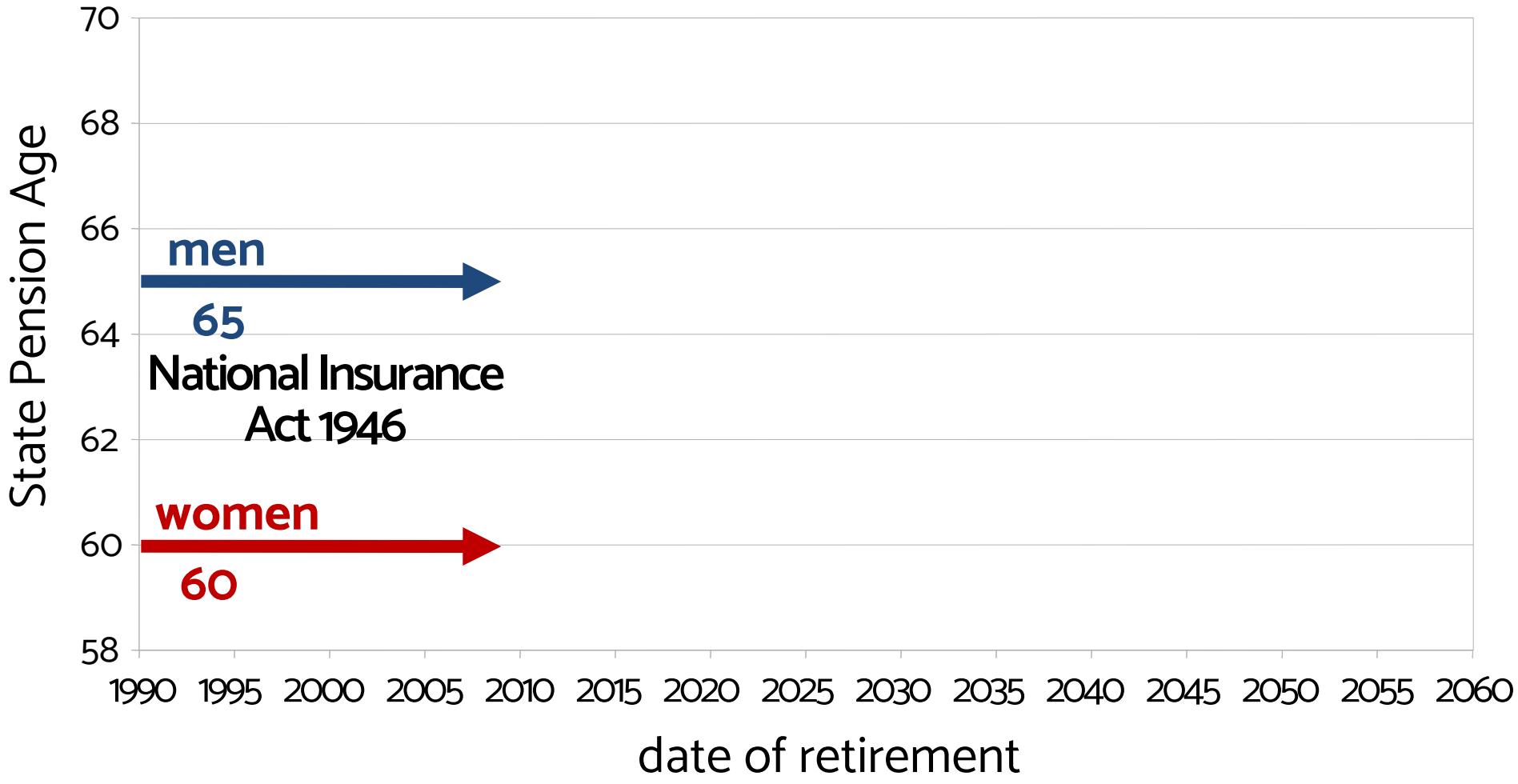
State pension system reforms



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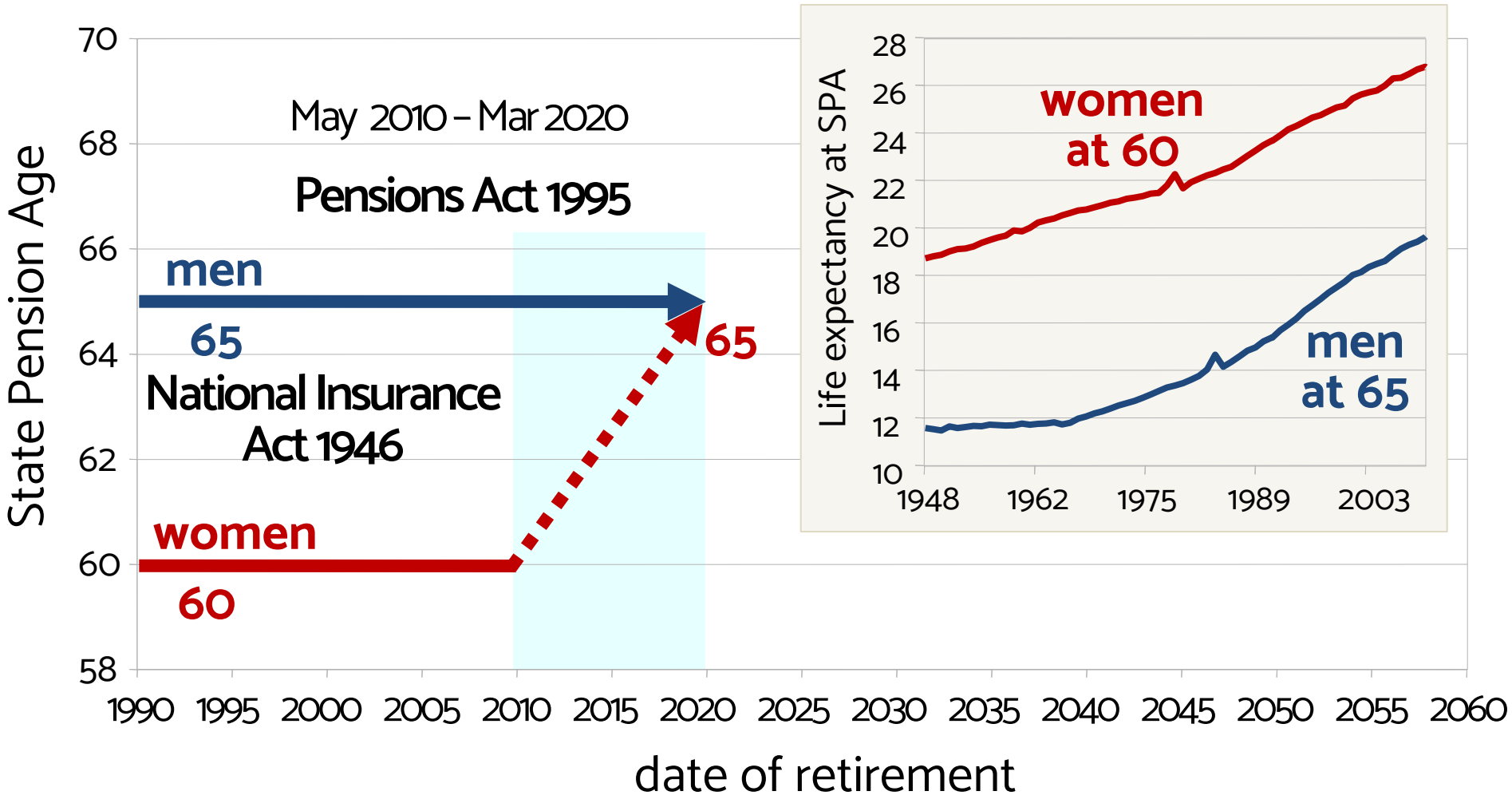

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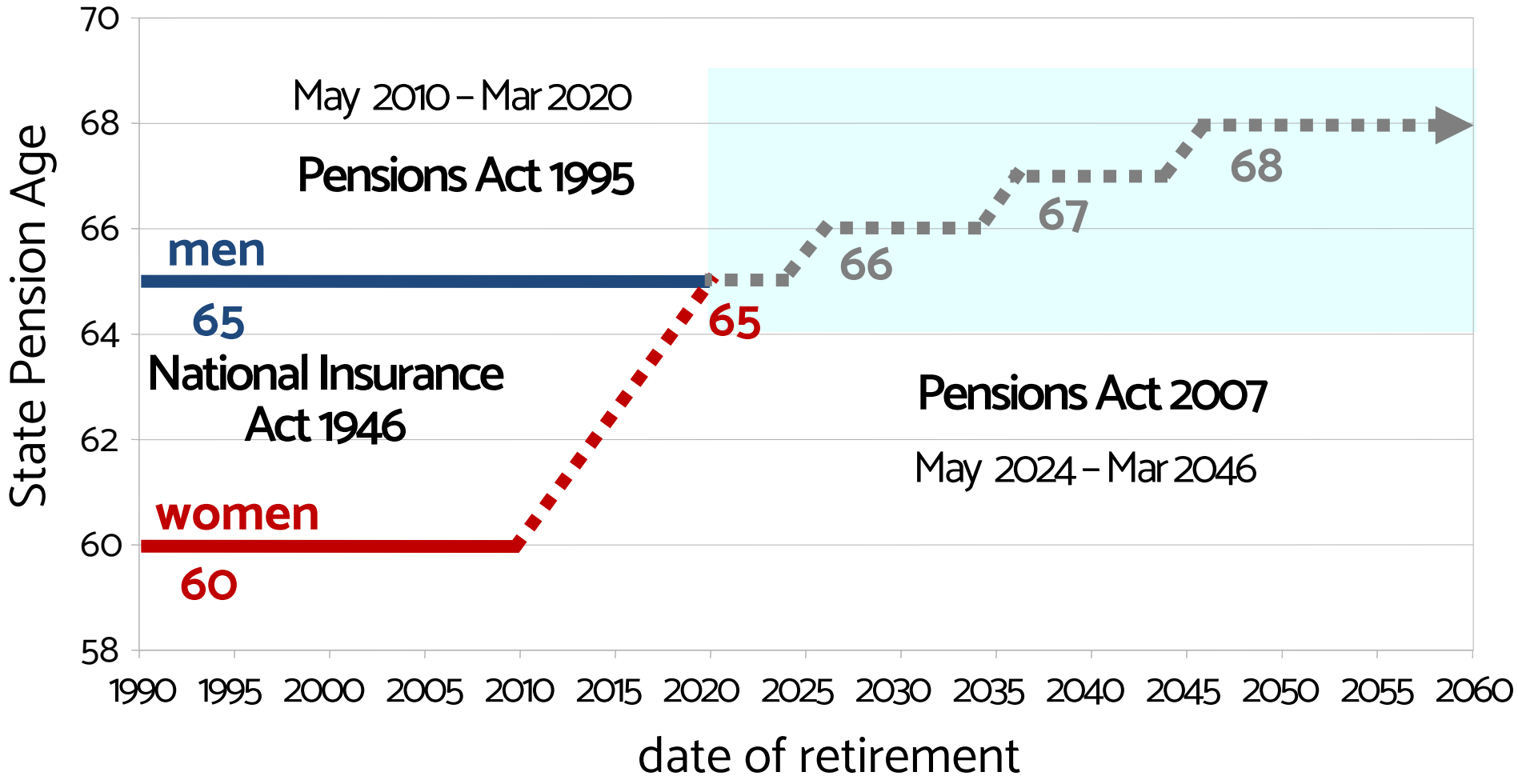

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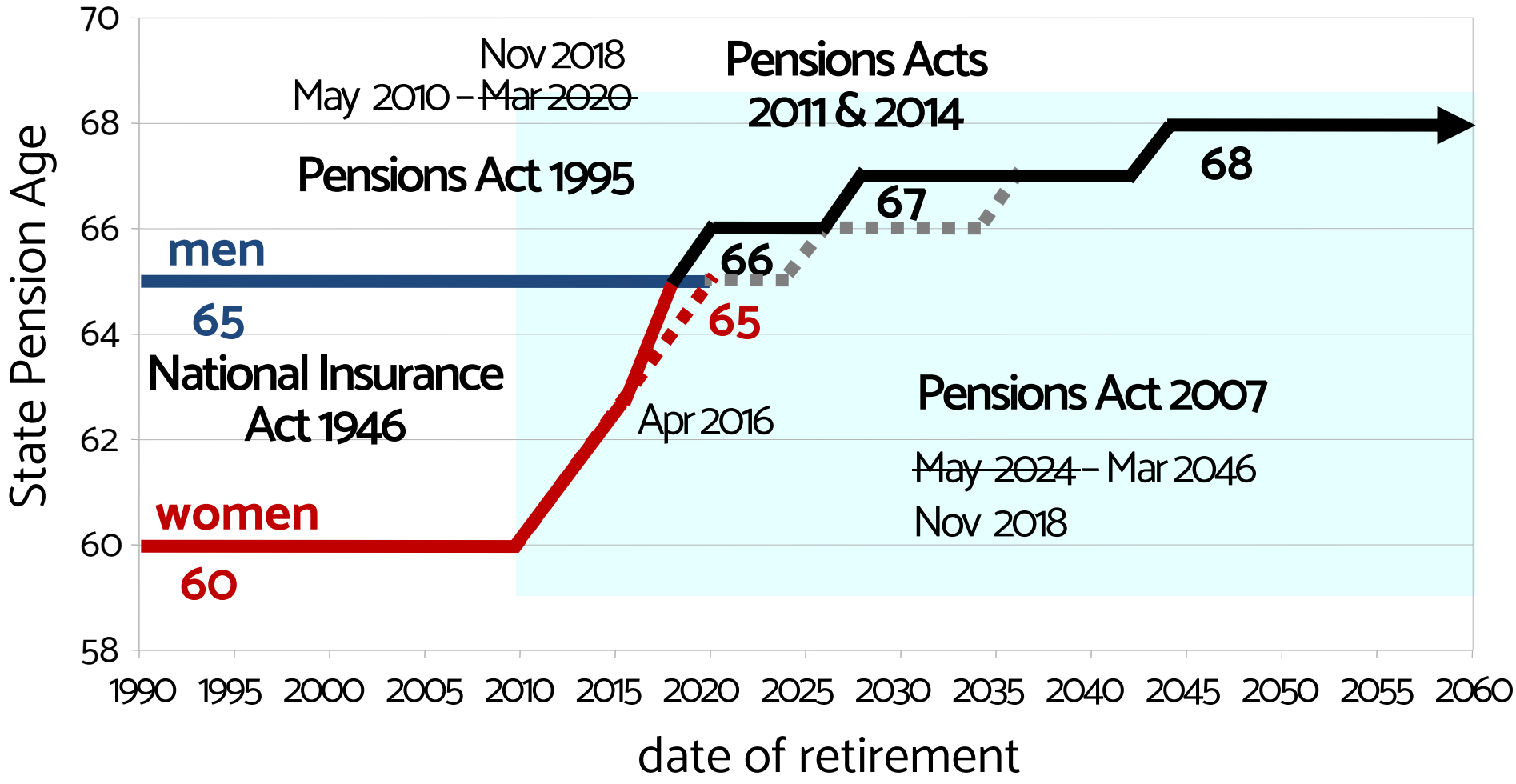

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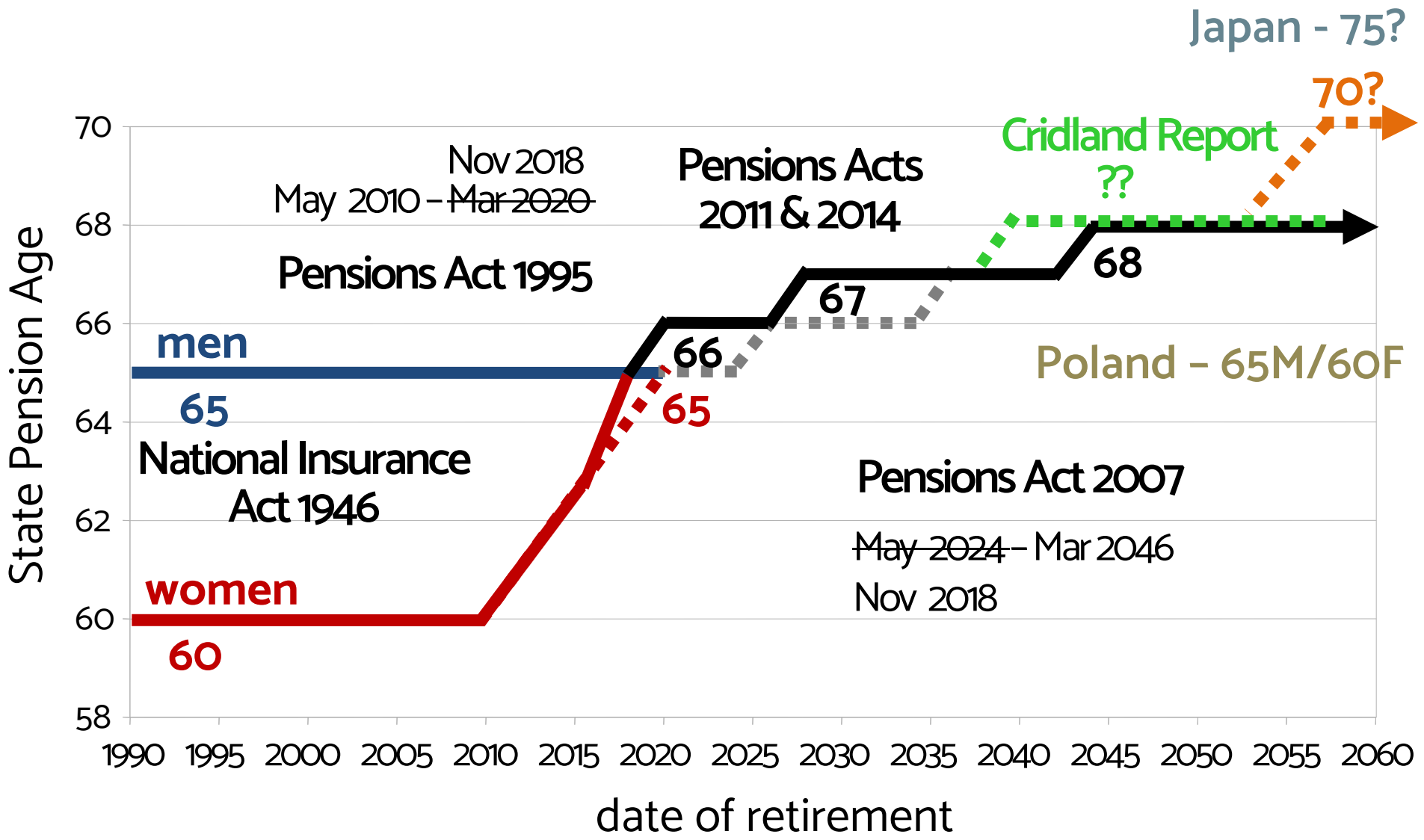
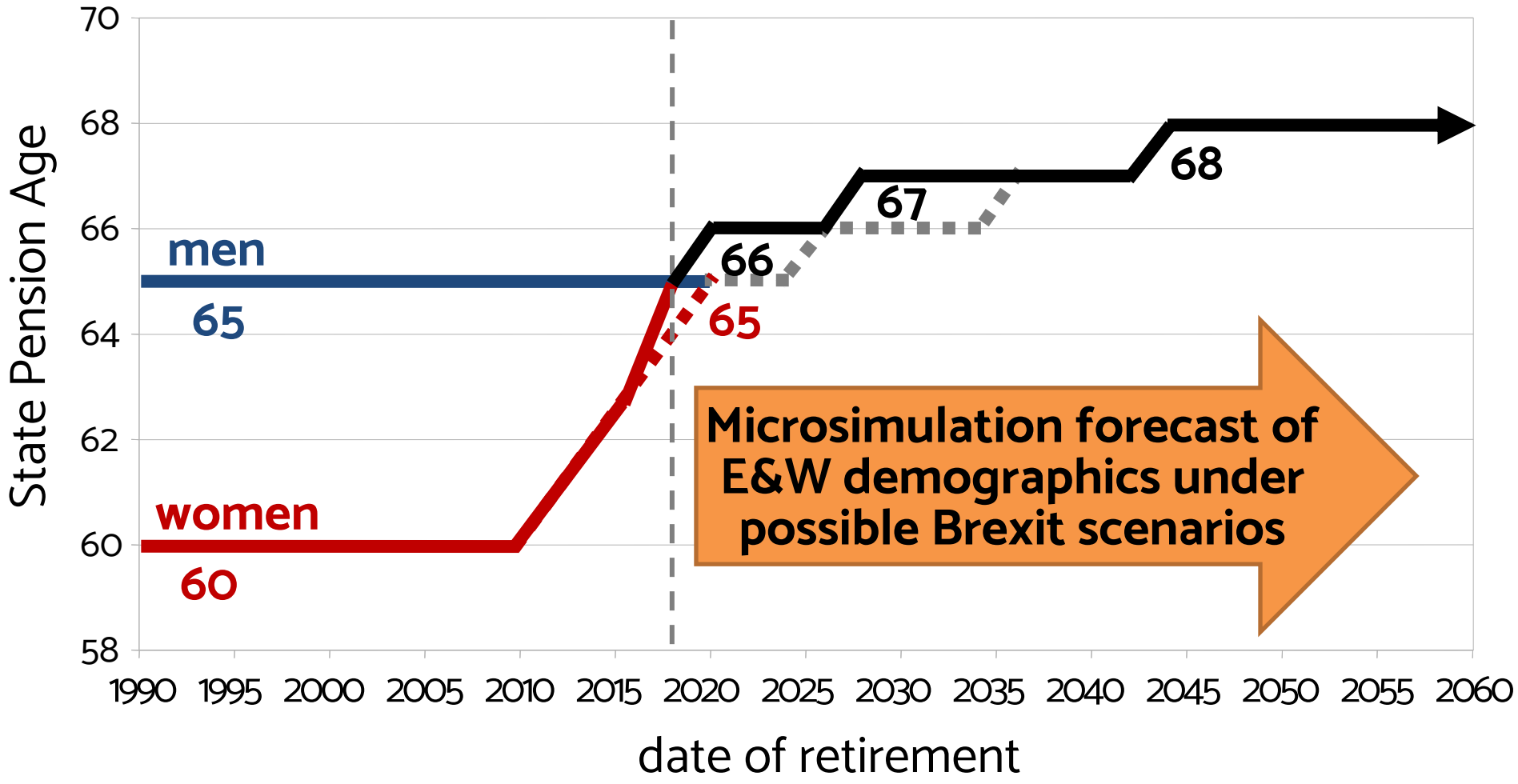


[Department for Work and Pensions, UK, 2014]

Forecasting the impact of state pension reforms in post-Brexit E&W



Microsimulation engine

stochastic microdynamics of complex adaptive systems

General purpose microsimulation engine

- **modern, object-oriented C++**
  - abstract interfaces & inheritance
  - encapsulation of private data
  - functional C++ for simpler & faster code
- **transparent API to extend the codebase by new kinds of**
  - agents (e.g. companies, animals, computers)
  - variables & interactions (e.g. BMI, marriage, loan)
  - stochastic processes (e.g. disease, education), ...
- **"observer API" for collecting & processing results**

Microsimulation

of the British population

PERSON

- age (date of birth DD/MM/YYYY)
- sex (male or female)
- pregnancy history, multiplicity (9M + 3M postpartum infertility)
- mother-children links (< 15-year-old migrating with mother)
- international migration history
- ethnicity ("inherited" from mother)
- mortality (forecast by neural network)

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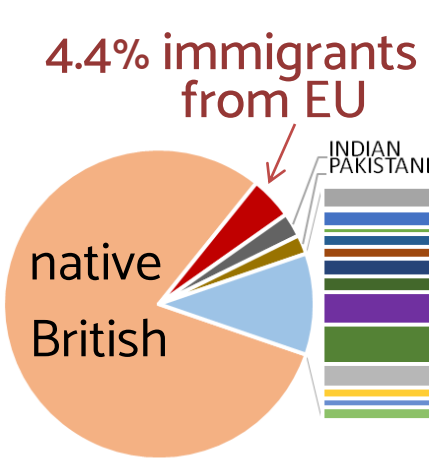
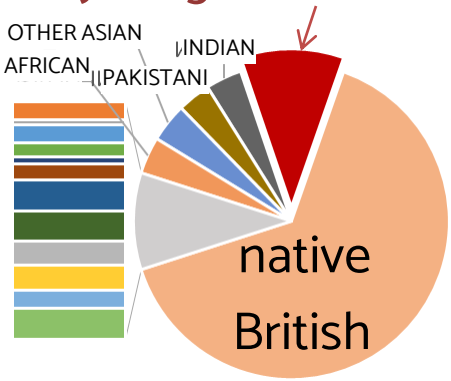


19.5% immigrants
UK Census **2011**

Polish Resettlement Act 1947
British Nationality Act 1948
*EU Directive on **free movement** 2004*

BREXIT: 3.6%
no BREXIT: 10.6%

EU immigrants
≅ young workers



20 year forecast

Int. J. of Microsimulation, 10(2), 2017

~30% by 2036

Microsimulation

of the British population

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Microsimulation

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- international migration history
- ethnicity ("inherited" from mother)
- **mortality (forecast by neural networks)**

  - "one in every three [children born in 2016] will live to see their 100th birthday", Office for National Statistics (1/2017)
  - "within the next 20 years, the average life expectancy in the developed world will rise to between 110 and 120", The Spectator (10/2017)
  - "Could humans live to 1,000 years old – and would we want to?", iNews (10/2017)
  - "Life expectancy in Britain has stagnated, meaning that a million years of life could disappear by 2058 – why?", The Independent (12/2017)
  - "Worsening outlook for UK life expectancy boosts insurers", Financial Times (3/2018)

Microsimulation

of the British population

PERSON

- age (date of birth DD/MM/YYYY)
- sex (male or female)
- pregnancy history, multiplicity (9M + 3M postpartum infertility)
- mother-children links (< 15-year-old migrating with mother)
- international migration history
- ethnicity ("inherited" from mother)
- mortality (forecast by neural networks)
- **fertility (forecast by neural networks)**

- ❑ overall decrease of fertility rates since 1930s, Office for National Statistics
- ❑ individual trends in different age groups, Office for National Statistics
- ❑ influence of cultural and religious background (Dormon, 2014; Dubuc&Haskey, 2010)
- ❑ fertility convergence across ethnic groups (e.g. Rees, 2007; Coleman & Smith, 2005)
- ❑ drop in pregnancies to predict recessions, Financial Times (2/2018)
- ❑ following socioeconomic trends with a 2 year lag, Eurostat 2013

Forecasting fertility & mortality rates with deep learning

Data: $q_{t,i}$ = qx mortality rate [ONS]

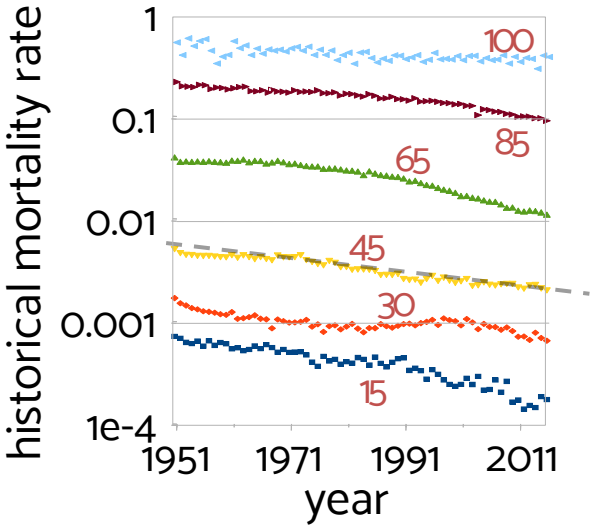
probability that a person alive at time t and of age i will die within the next year

age groups i = 0, 1, ... 100+

years t = 1951–2016

two datasets (men & women)

$$x_{t,i} = \ln q_{t,i}$$



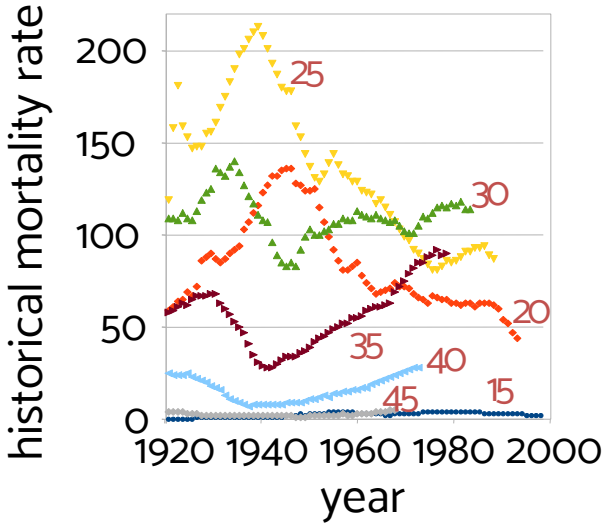
Data: fertility rate [ONS]

number of births per 1000 women in each age group that had a birth in each year

childbearing age i = 15, 16, ... 45+

birth cohorts t = 1920–1999

(weighted by ethnic groups)



Goal: discover complex patterns & trends – without overfitting

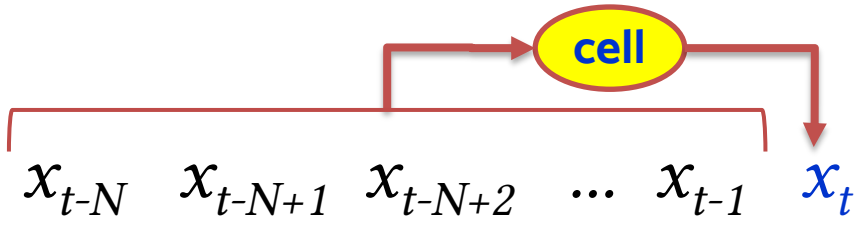
Method: RECURRENT NEURAL NETWORK

Forecasting mortality rates with deep learning

RECURRENT NEURAL NETWORK

$$x_{t,i} = \ln q_{t,i}$$

(log-qx mortality rate)



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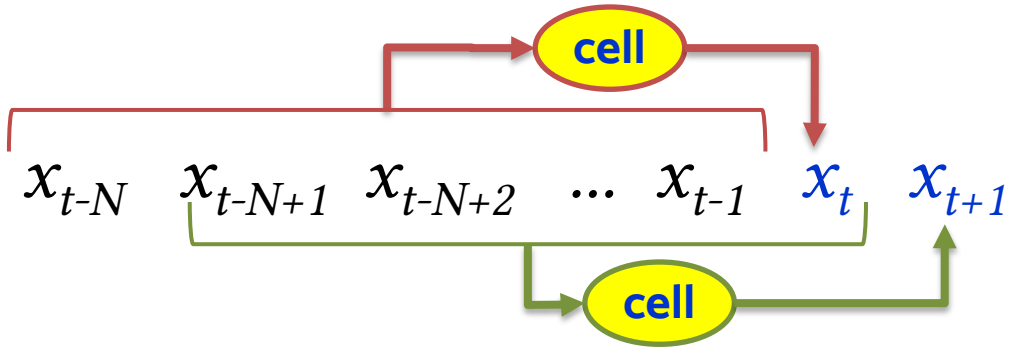

Forecasting mortality rates with deep learning

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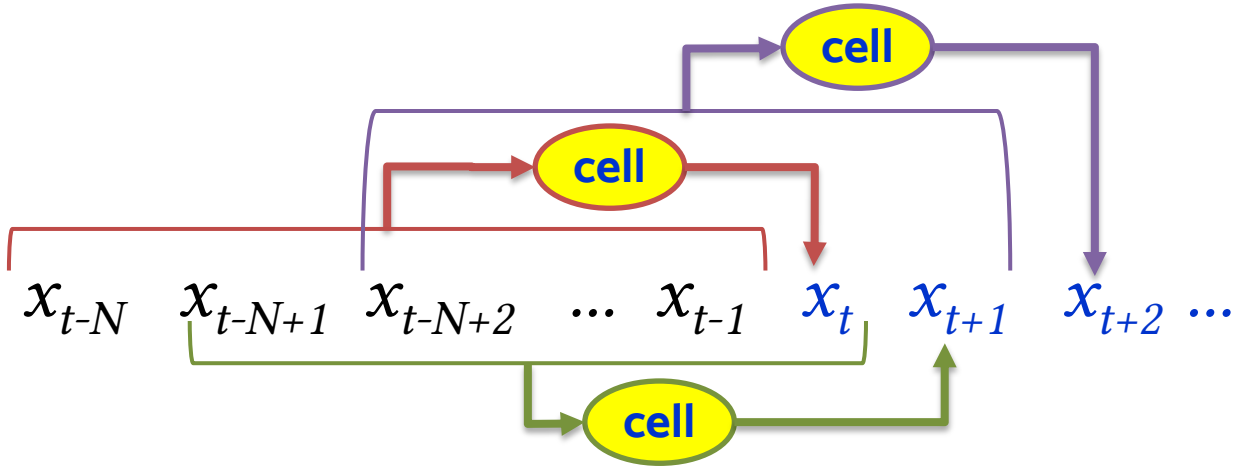

$$x_{t,i} = \ln q_{t,i}$$

(log-qx mortality rate)

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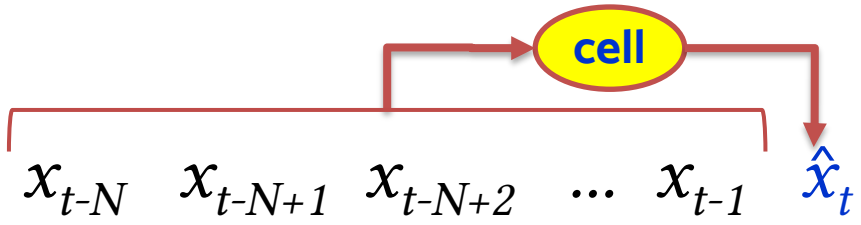

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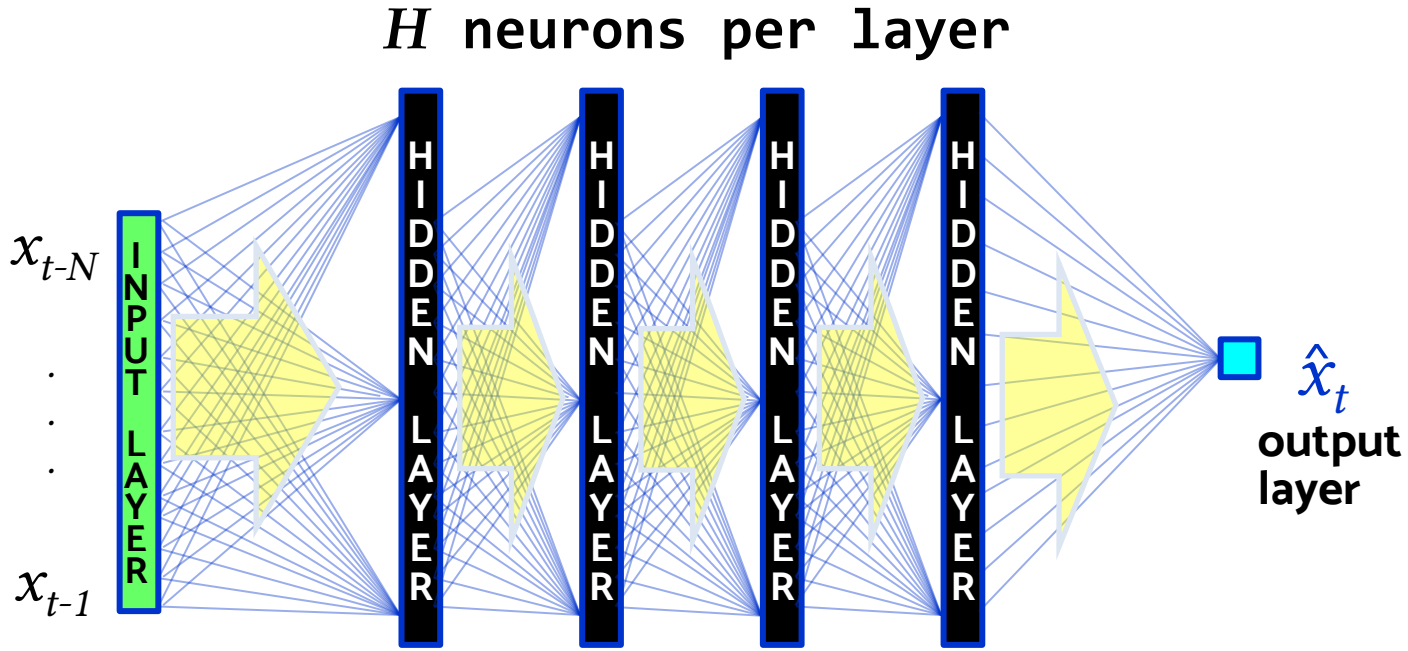
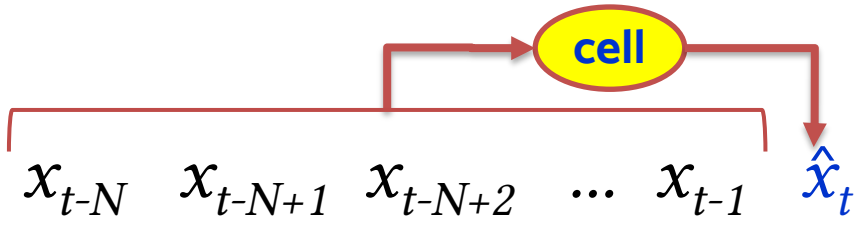


$$z_k = \sigma(W_k z_{k-1} + b_k)$$

parameters $\theta = \{\text{weights } W_k, \text{ biases } b_k\}$

σ - activation function ReLU

RECURRENT NEURAL NETWORK – TRAINING



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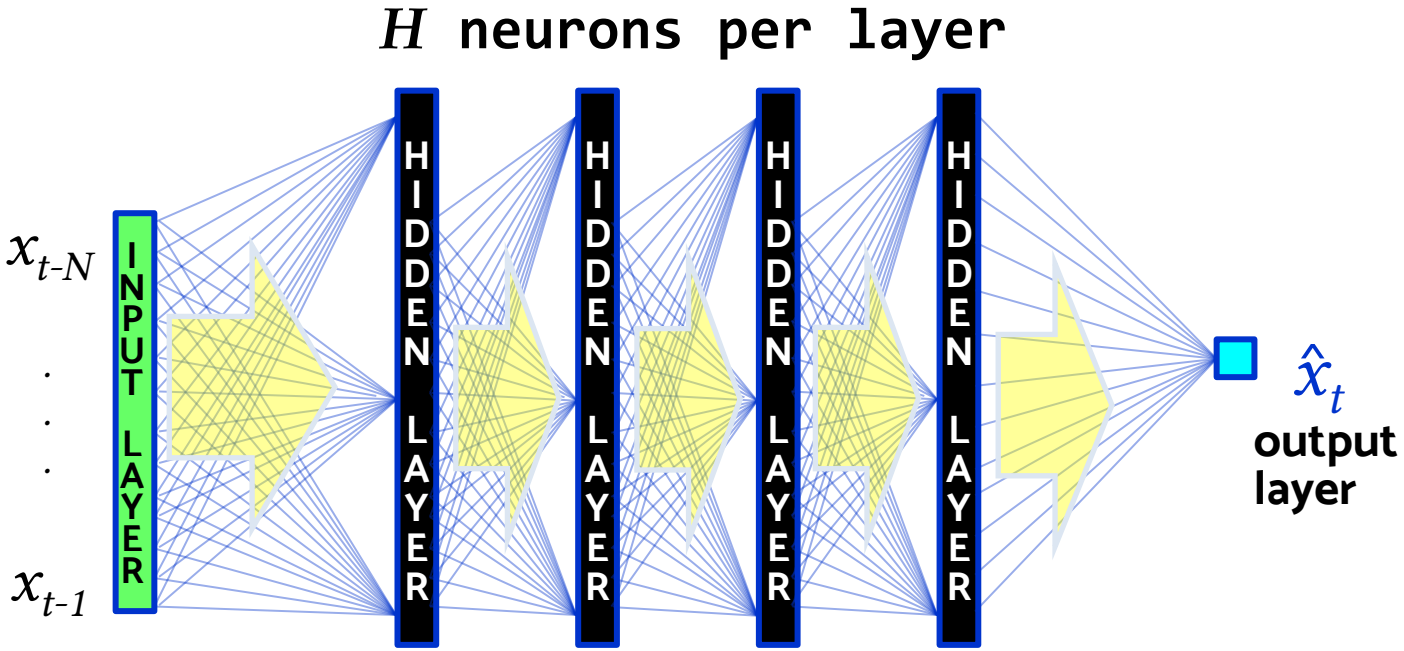

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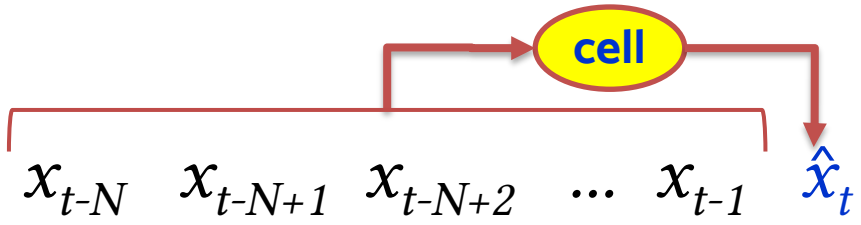

$$x_{t,i} = \ln q_{t,i}$$

(log-qx mortality rate)

error: $\hat{x}_t - x_t = \varepsilon_0$

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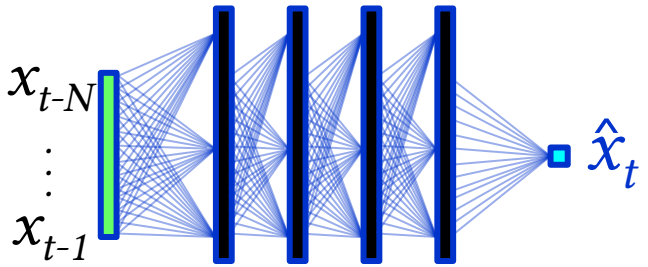

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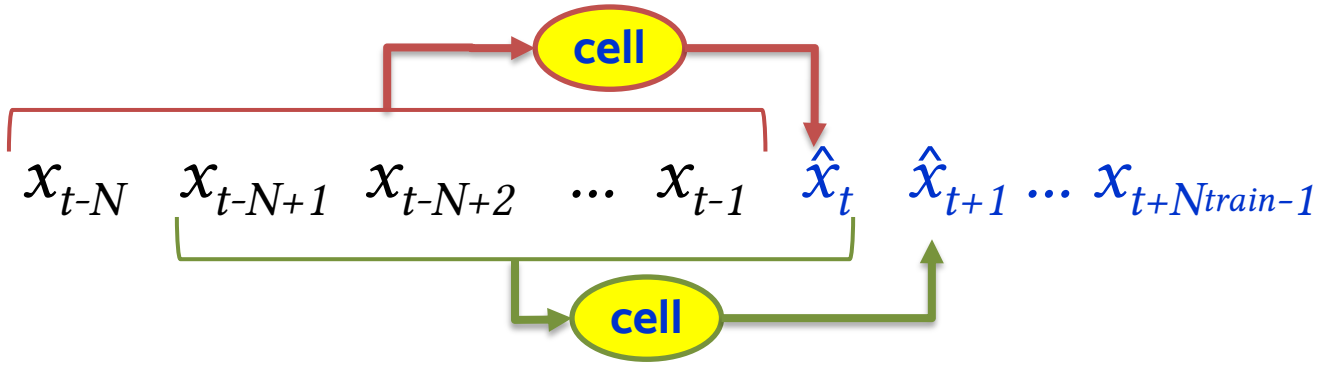

$$x_{t,i} = \ln q_{t,i}$$

(log-qx mortality rate)

error: $\hat{x}_t - x_t = \varepsilon_0$

$\hat{x}_{t+1} - x_{t+1} = \varepsilon_1$

… $\varepsilon_{Ntrain-1}$

loss function: $c[f] = \Sigma \varepsilon_n^2$

/ K-L divergence

$$z_k = \sigma(W_k z_{k-1} + b_k)$$

parameters $\theta = \{$weights W_k; biases $b_k\}$

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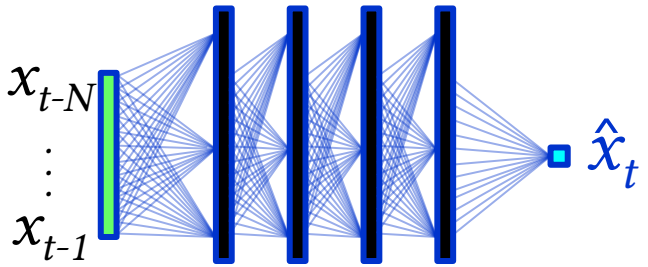

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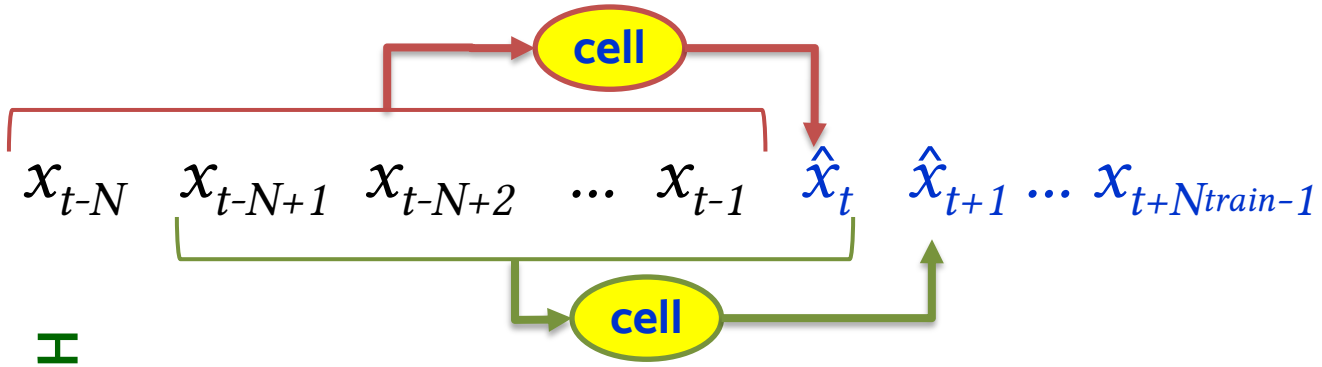

$$x_{t,i} = \ln q_{t,i}$$

(log-qx mortality rate)

error: $\hat{x}_t - x_t = \varepsilon_0$

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... $\varepsilon_{Ntrain-1}$

MINIBATCH

$x_{t'-N}$ $x_{t'-N+1}$ ... $x_{t'-1}$ $x_{t'}$ ... $x_{t'+Ntrain-1}$

$x_{t''-N}$ $x_{t''-N+1}$ ... $x_{t''-1}$ $x_{t''}$ ... $x_{t''+Ntrain-1}$

...

$x_{t'''-N}$ $x_{t'''-N+1}$ ... $x_{t'''-1}$ $x_{t'''}$ ... $x_{t'''+Ntrain-1}$

ε' ε'' ε'''

loss function: $c[f] = \sum \varepsilon_n^2$

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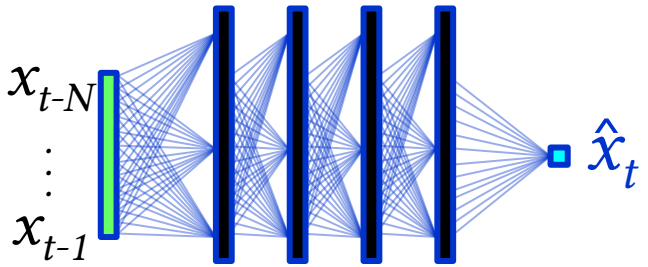

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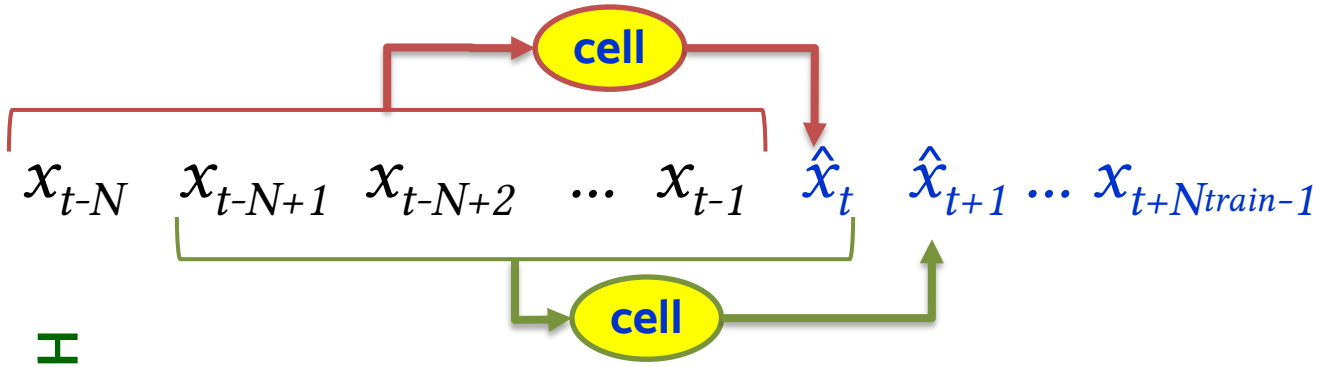

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$x_{t'''-N}$ $x_{t'''-N+1}$ … $x_{t'''-1}$ $x_{t'''}$ … $x_{t'''+Ntrain-1}$

from training set

ε' ε'' ε'''

loss function: $c[f] = \sum \varepsilon_n^2$

backpropagation:

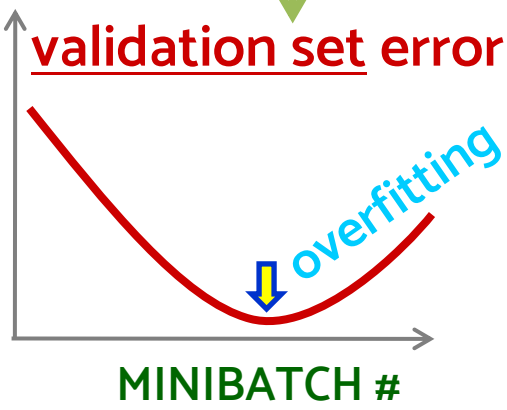
$$\theta \rightarrow \theta - \alpha \nabla_\theta c[f]$$

stoch. gradient descent

NEW MINIBATCH

(and so on ~300,000 times)

validation set error



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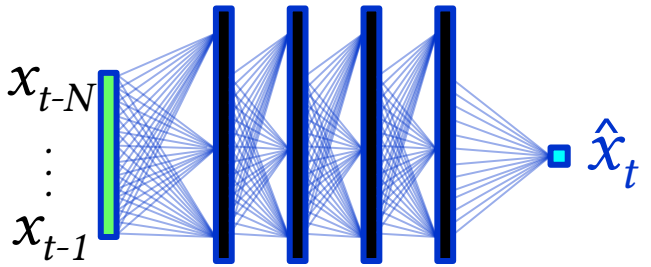

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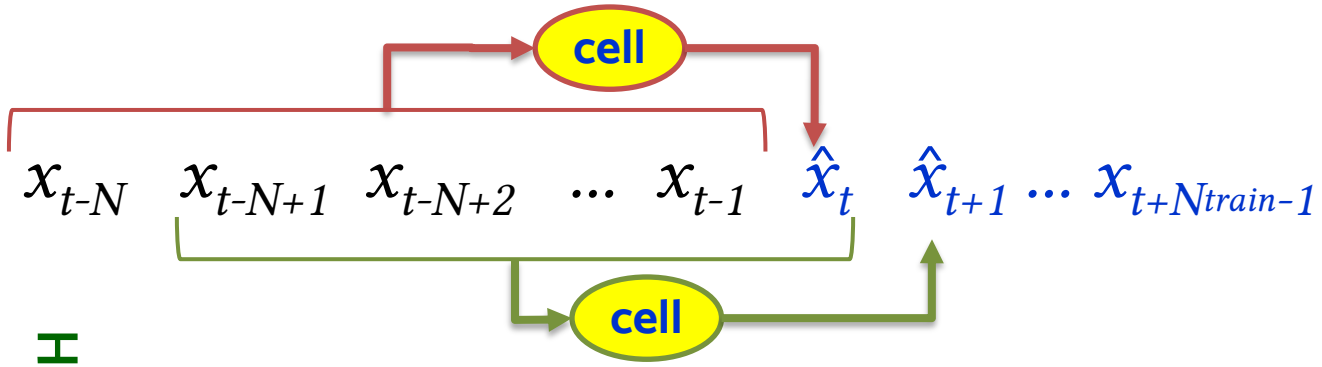

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... $\varepsilon_{Ntrain-1}$

MINIBATCH

$x_{t'-N}$ $x_{t'-N+1}$ ... $x_{t'-1}$ $x_{t'}$ ... $x_{t'+Ntrain-1}$

$x_{t''-N}$ $x_{t''-N+1}$ ... $x_{t''-1}$ $x_{t''}$ ... $x_{t''+Ntrain-1}$

...

$x_{t'''-N}$ $x_{t'''-N+1}$ ... $x_{t'''-1}$ $x_{t'''}$ ... $x_{t'''+Ntrain-1}$

from training set

ε' ε'' ε'''

loss function: $c[f] = \sum \varepsilon_n^2$

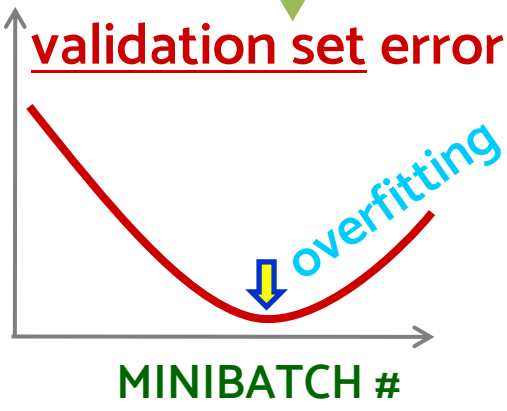
backpropagation:

$\theta \rightarrow \theta - \alpha \nabla_\theta c[f]$

stoch. gradient descent

NEW MINIBATCH

(and so on ~300,000 times)

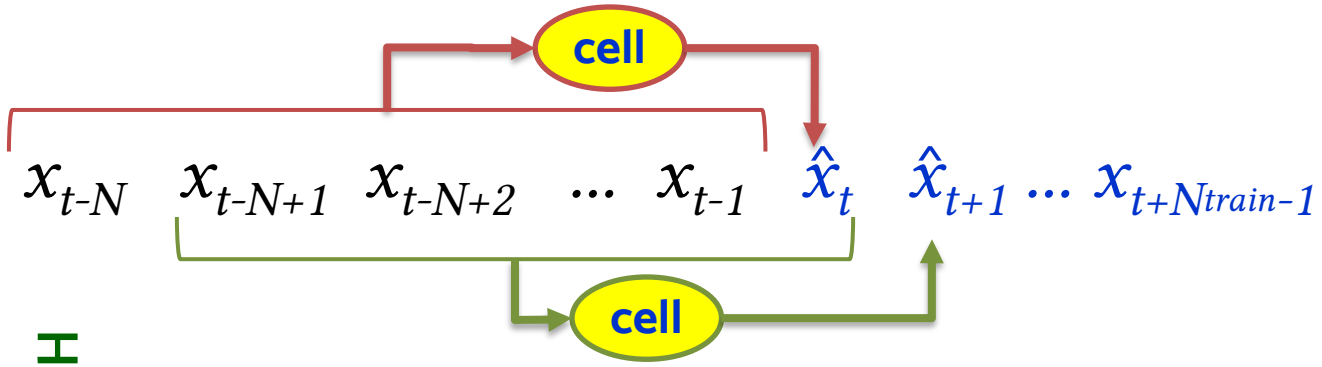


OPTIMISATION

finding RNN hyperparameters

nr of layers/neurons, input size, ...

RECURRENT NEURAL NETWORK – TRAINING



$$x_{t,i} = \ln q_{t,i}$$

(log-qx mortality rate)

error: $\hat{x}_t - x_t = \varepsilon_0$

$\hat{x}_{t+1} - x_{t+1} = \varepsilon_1$

... $\varepsilon_{Ntrain-1}$

MINIBATCH

$x_{t'-N}$ $x_{t'-N+1}$ ... $x_{t'-1}$ $x_{t'}$ ... $x_{t'+Ntrain-1}$

$x_{t''-N}$ $x_{t''-N+1}$ ... $x_{t''-1}$ $x_{t''}$ ... $x_{t''+Ntrain-1}$

$x_{t'''-N}$ $x_{t'''-N+1}$ ... $x_{t'''-1}$ $x_{t'''}$ ... $x_{t'''+Ntrain-1}$

from training set

ε' ε'' ε'''

loss function: $c[f] = \sum \varepsilon_n^2$

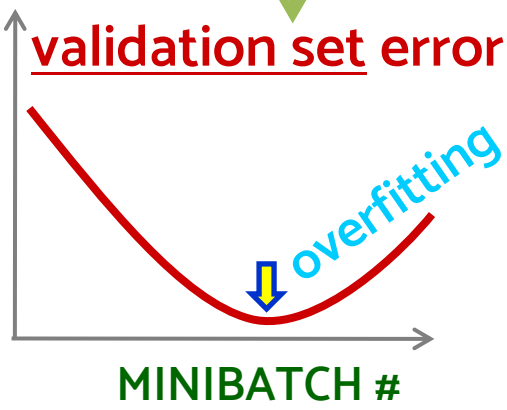
backpropagation:

$\theta \rightarrow \theta - \alpha \nabla_\theta c[f]$

stoch. gradient descent

NEW MINIBATCH

(and so on ~300,000 times)



OPTIMISATION

finding RNN hyperparameters

nr of layers/neurons, input size, ...

When you look into data, the data looks into you!

FINAL TEST

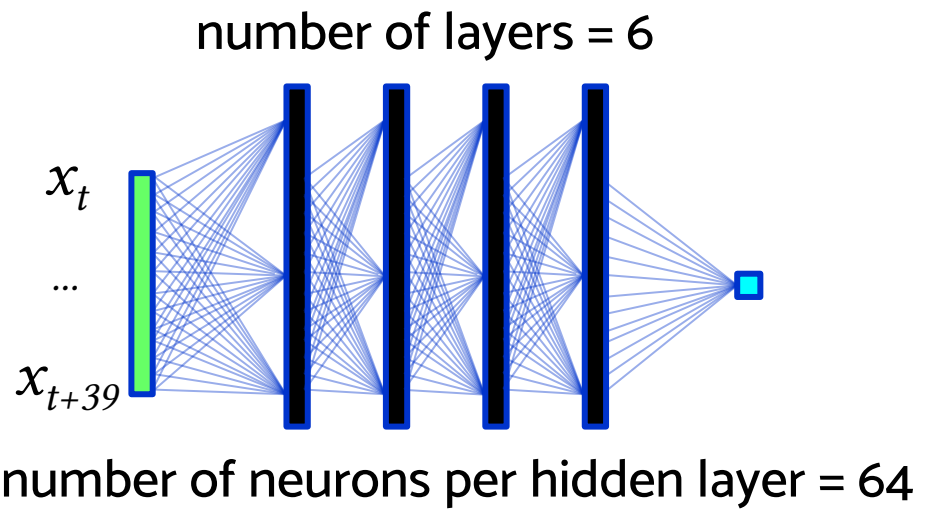
- train on training+validation set
- calculate error on test set

RECURRENT NEURAL NETWORK – APPLICATION

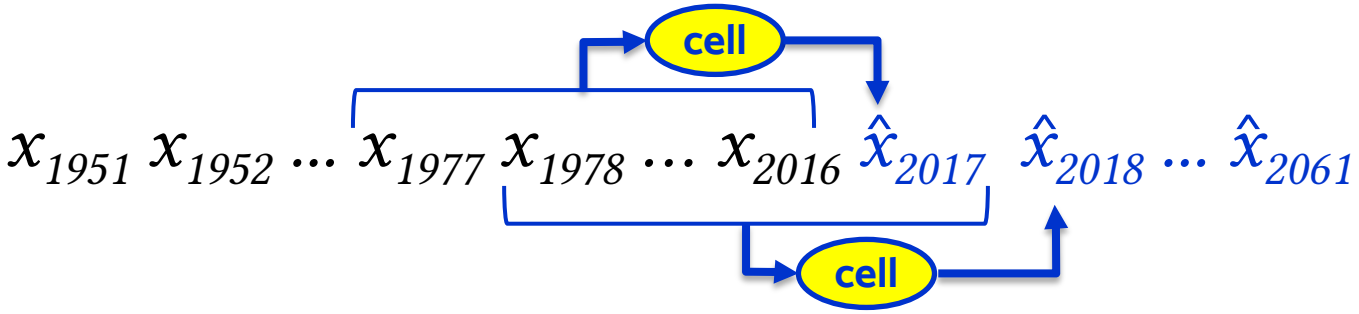
$$x_{t,i} = \ln q_{t,i}$$

(log-qx mortality rate)

- optimised RNN:



- re-train on full dataset & extrapolate for each age group i:

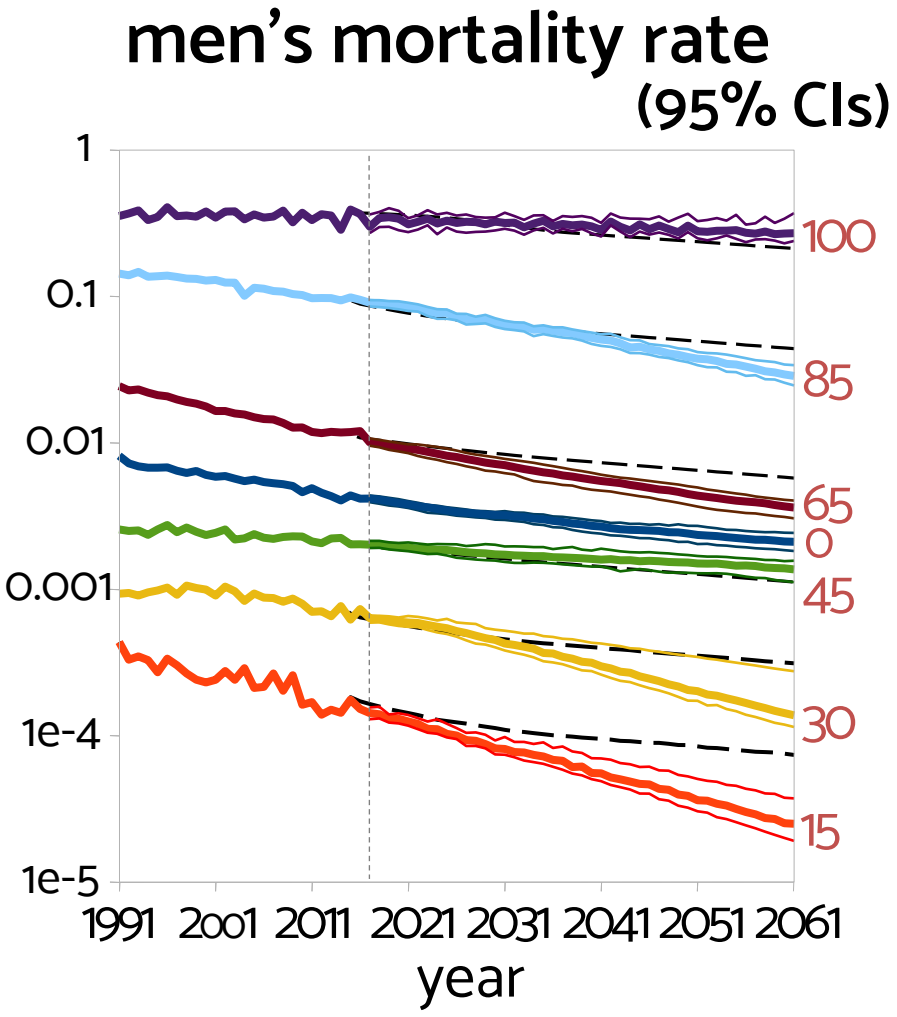


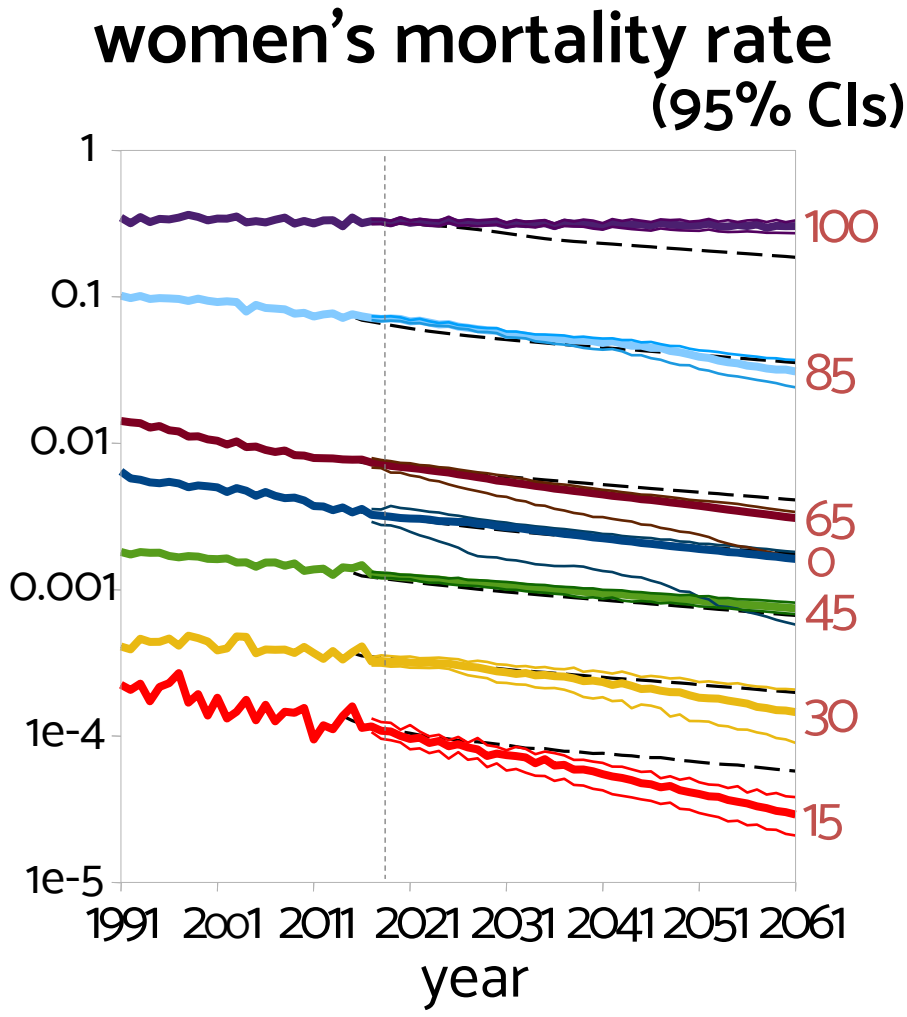
- repeat 100 times:

mortality = median value

CIs = top & bottom percentiles

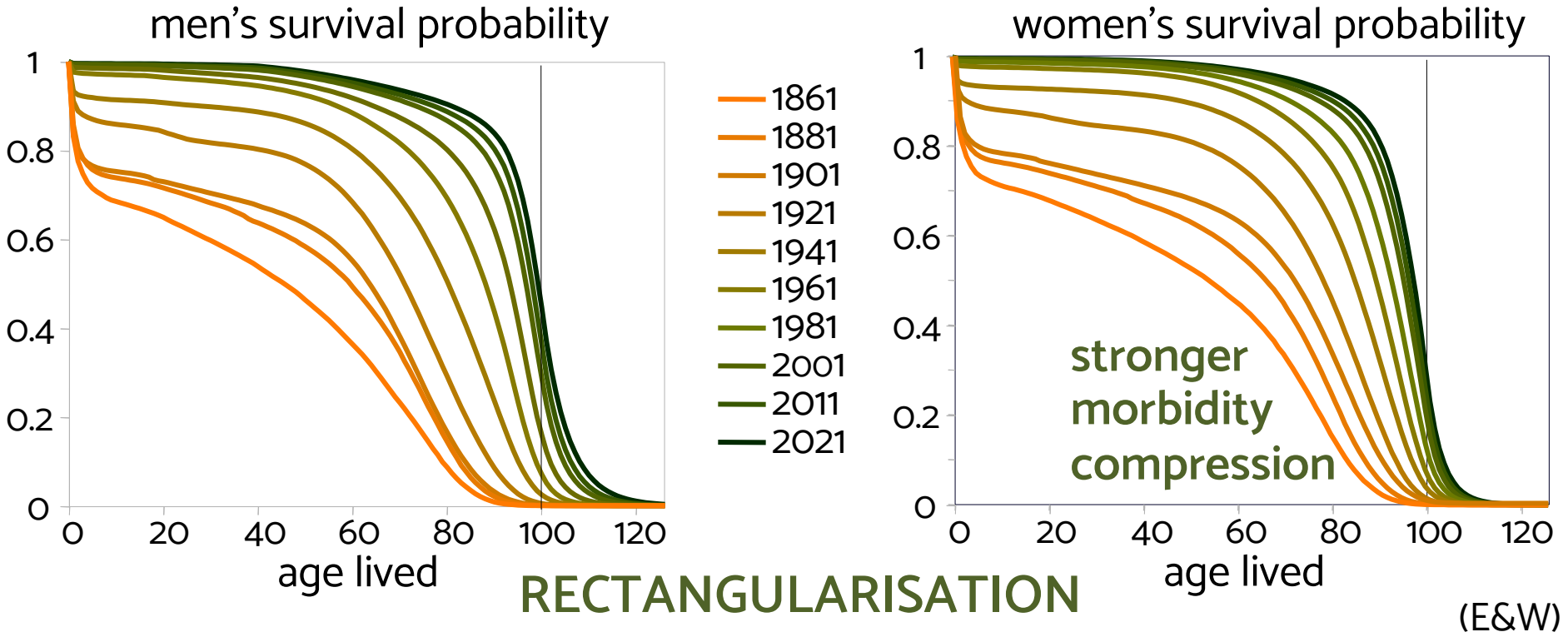
Mortality rate forecast by RNN





– – – ONS projections for E&W

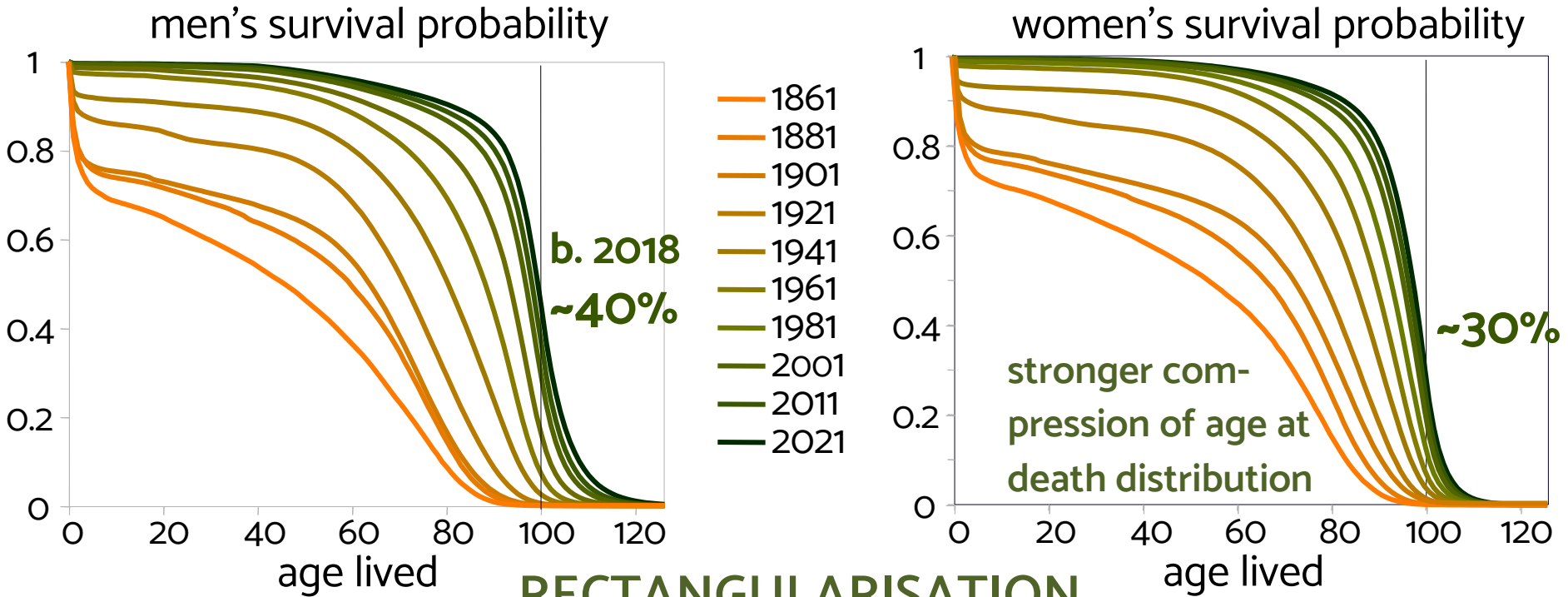
Survival curve forecast by RNN



RECTANGULARISATION

(E&W)

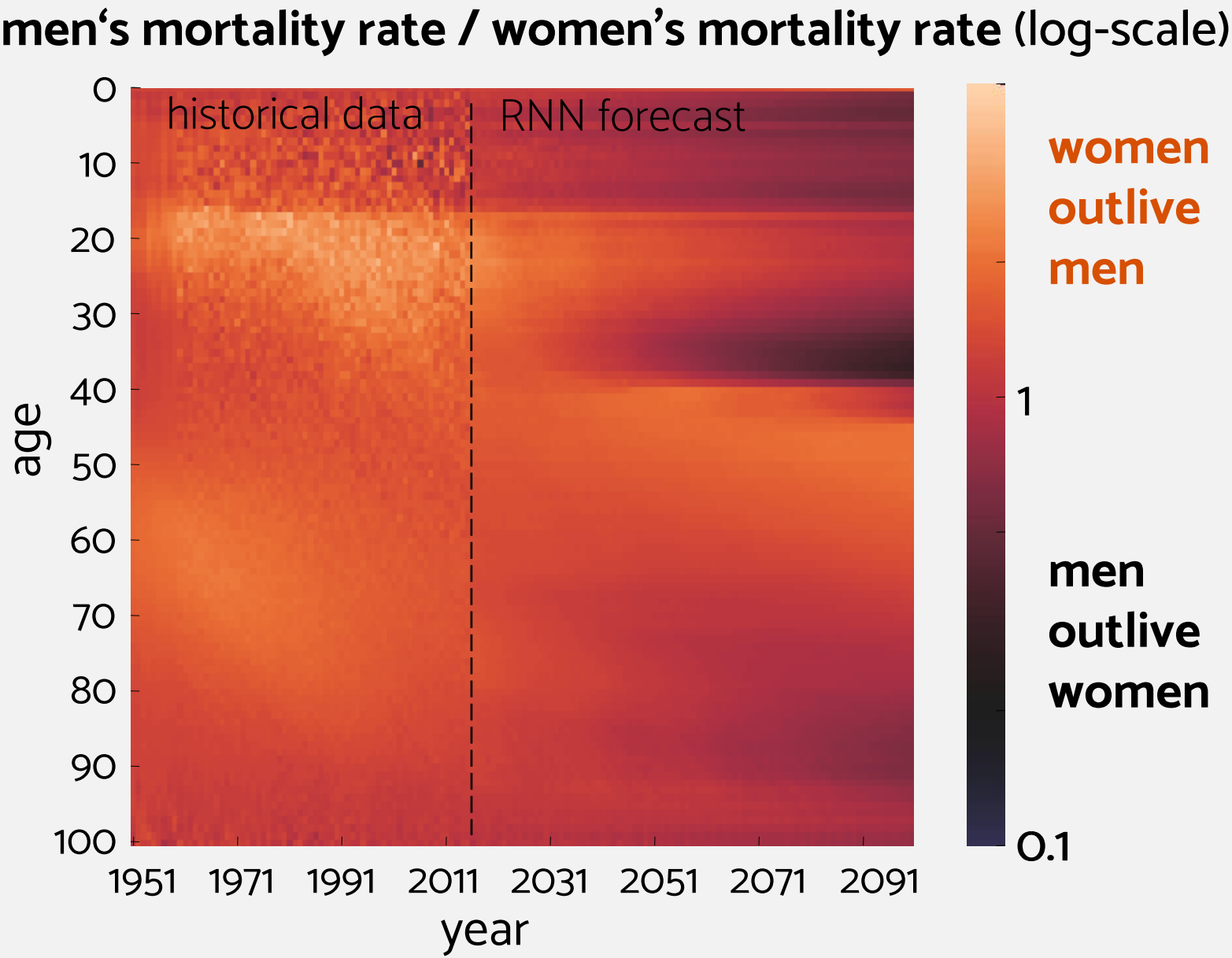
Survival curve forecast by RNN



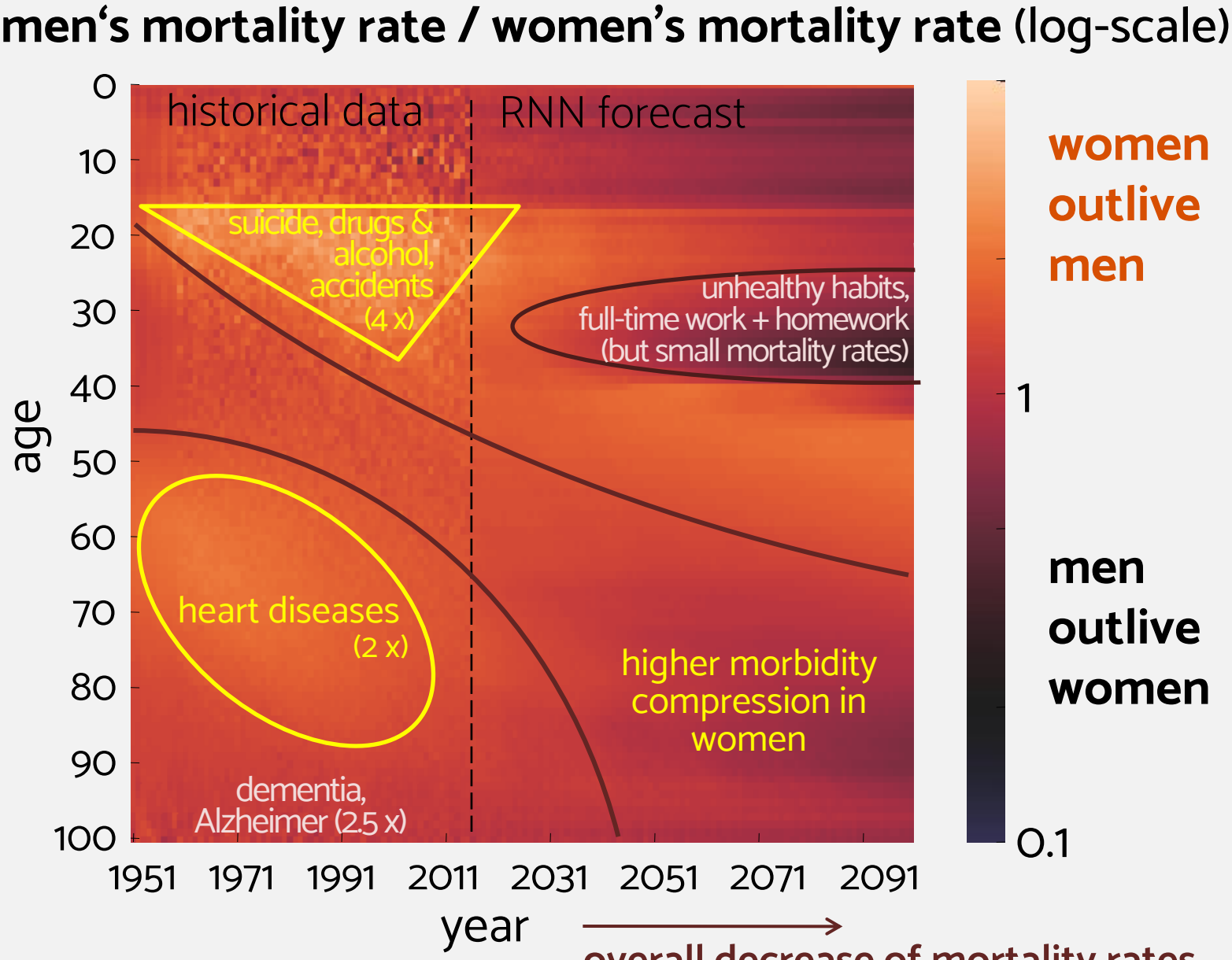
RECTANGULARISATION

- ONS (2014): "There is a long-running trend since the 1970s for male life expectancy to catch up with female, and in some areas they have now caught or surpassed it."
- ONS (2015): "Most affluent man outlives the average woman for the first time"
- PHE (2014): men can expect to outlive women in 110 districts in England; biggest gap in Bewbush & Broadfield: men 96 / women 83

How men's lifespan outpaces women's?

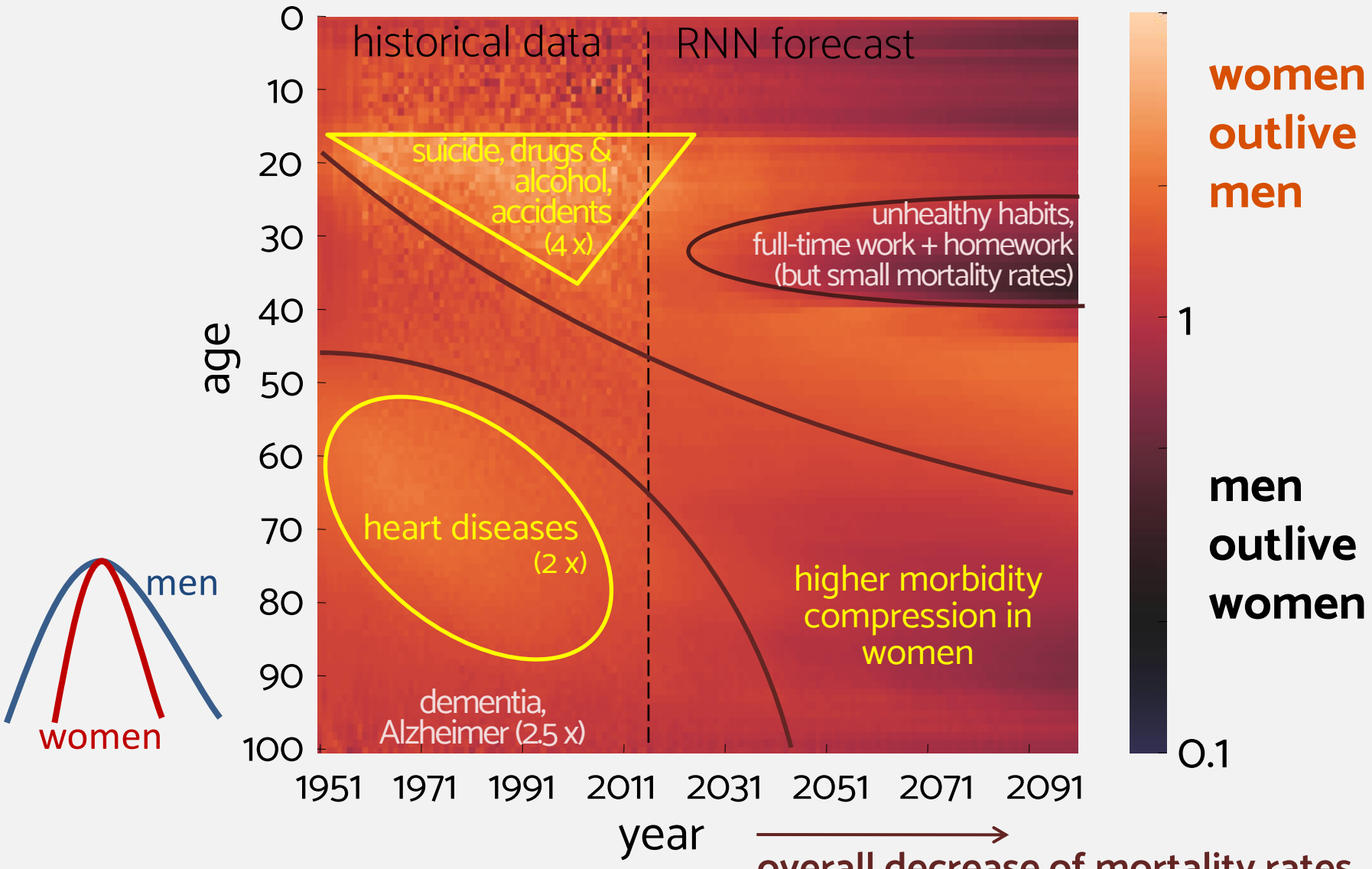


Deep learning – hidden insight into data?



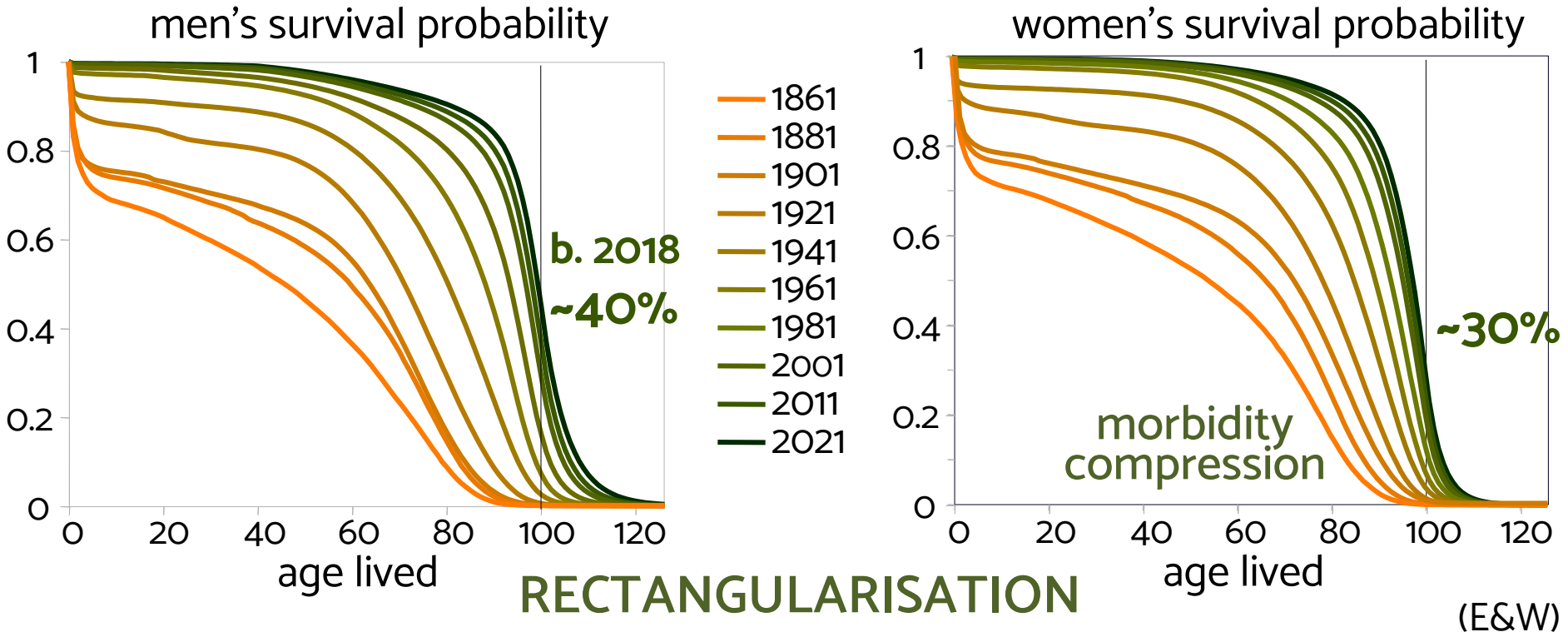
Causes of death by age [ONS]

Deep learning – hidden insight into data?



Causes of death by age [ONS]

Survival curve forecast by RNN

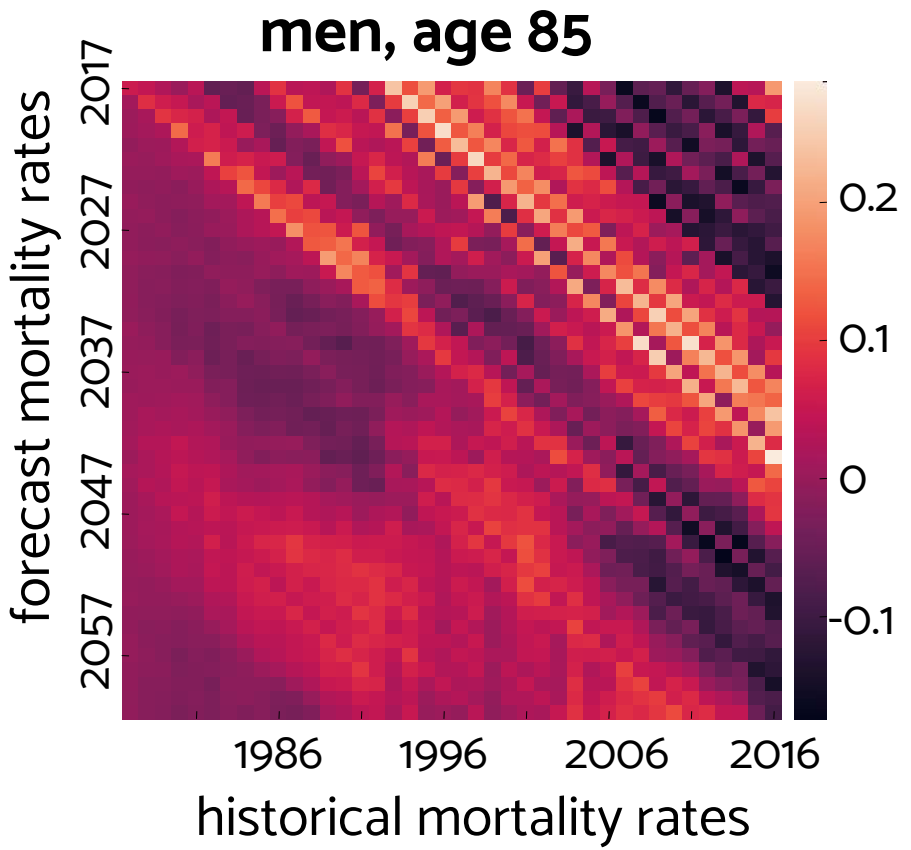


Life expectancy at SPA (for women / for men)

Year	60 W / 65 M	Equal age (65)	Current reform (68)
1948	24% **18.7 / 11.6** 15%	14.5 / 11.6	12.3 / 9.8
2018	28.5 / 22.2	26% **23.4 / 22.2** 27%	20.7 / 19.3
2048	31.8 / 27.9	26.8 / 27.9	26% **23.8 / 24.8** 25%

Sensitivity analysis

Sensitivity of forecast mortality rates to historical ones: $\partial q_{t,i} \Big/ \partial q_{t',i}$



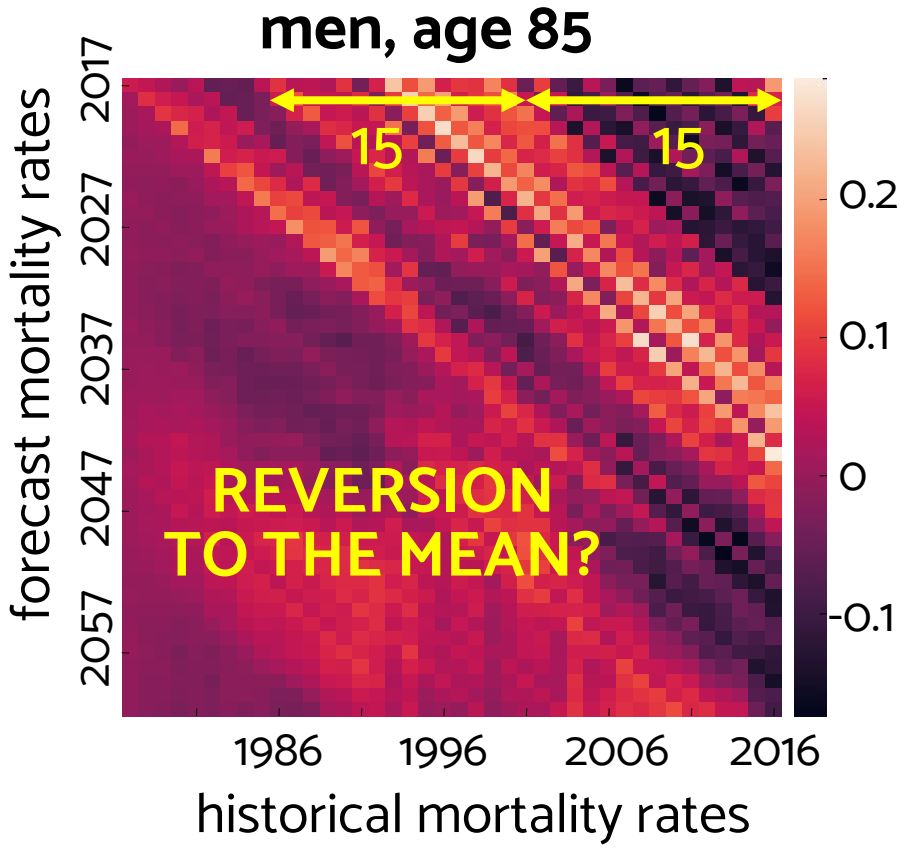
Sensitivity analysis

Sensitivity of forecast mortality rates to historical ones: $\partial q_{t,i} / \partial q_{t',i}$

(explanation)
Taylor series expansion:

$$q_t \approx \sum_{t'} \partial q_t / \partial q_{t'} \cdot q_{t'} + \text{const.}$$

$$\sum_{t'} \partial q_t / \partial q_{t'} \cdot q_{t'} \approx - \frac{(\overline{q_{\{t-1 \dots t-15\}}} - \overline{q_{\{t-16 \dots t-30\}}})}{30}$$

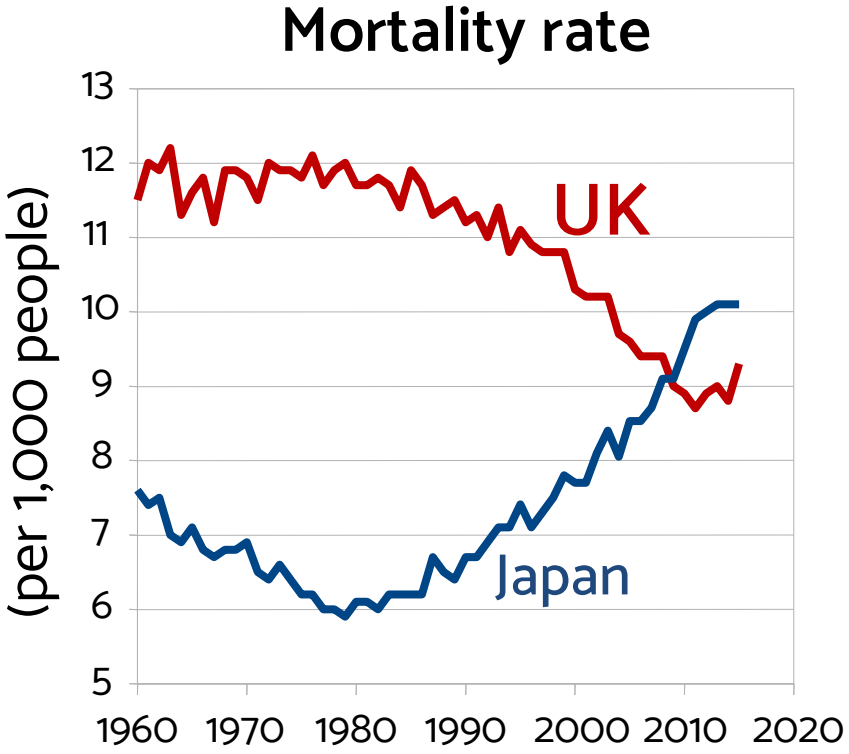


→ upward future trend?

Sensitivity analysis

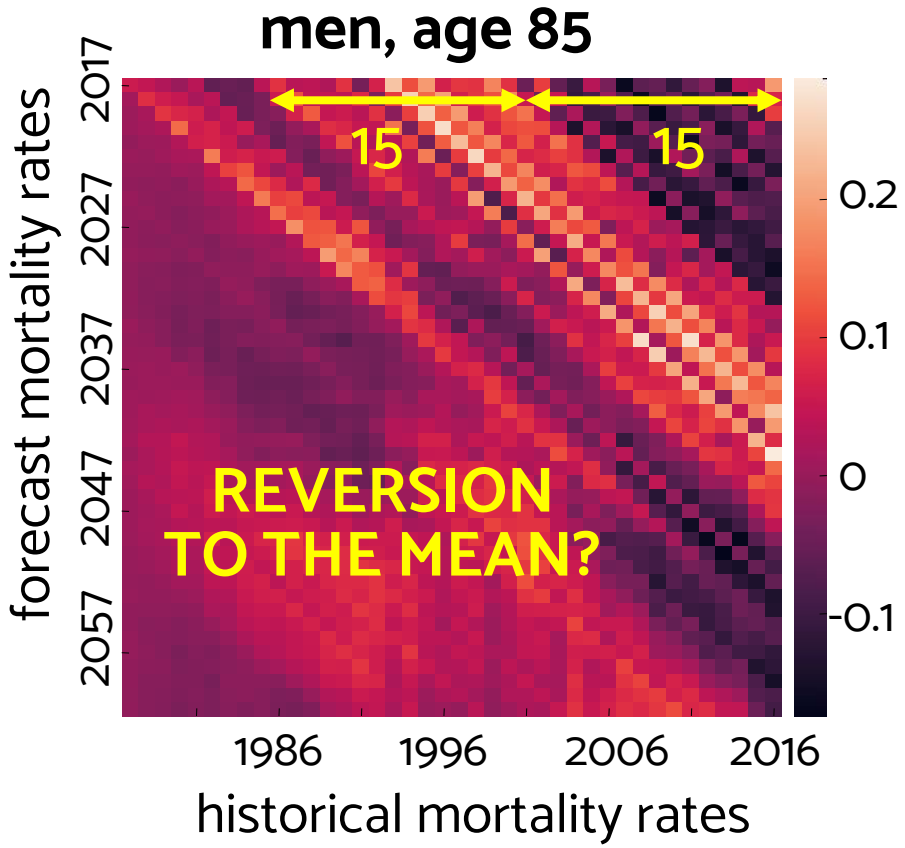
- "Average UK life expectancy falls", *Financial Times* (3/2017)
- "Life expectancy falls by a year in several regions of England", *The Times* (1/2018)
- "Life Expectancy In Britain Falling For The First Time In 110 years", *Newsweek* (11/2017)

[based on ONS reports]



→ upward future trend?

→ hallmark of survival bias?



Microsimulation

State pension reforms & Brexit (Britain's exit from EU)

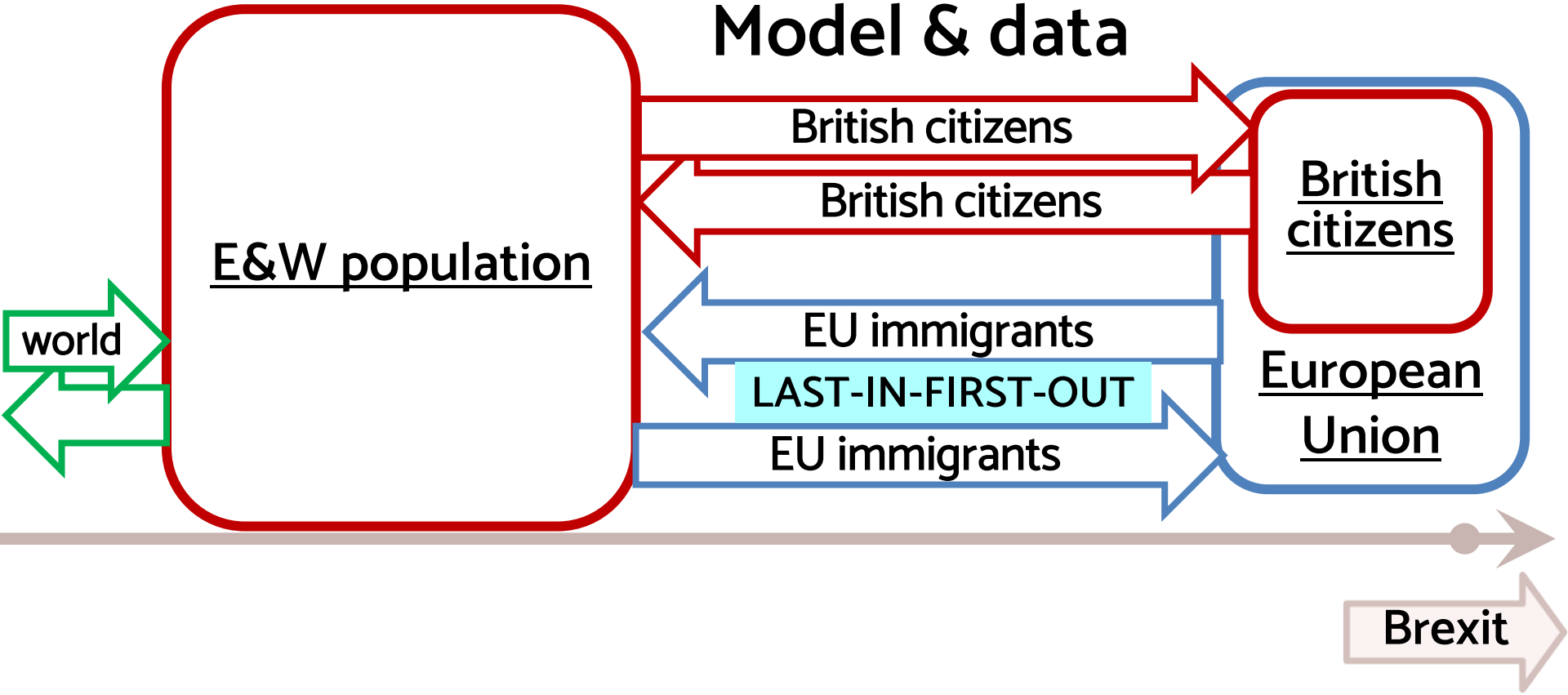
PERSON

- age (date of birth DD/MM/YYYY)
- sex (male or female)
- pregnancy history, multiplicity (9M + 3M postpartum infertility)
- mother-children links (< 15-year-old migrating with mother)
- international migration history
- ethnicity ("inherited" from mother)
- mortality (forecast by neural networks)

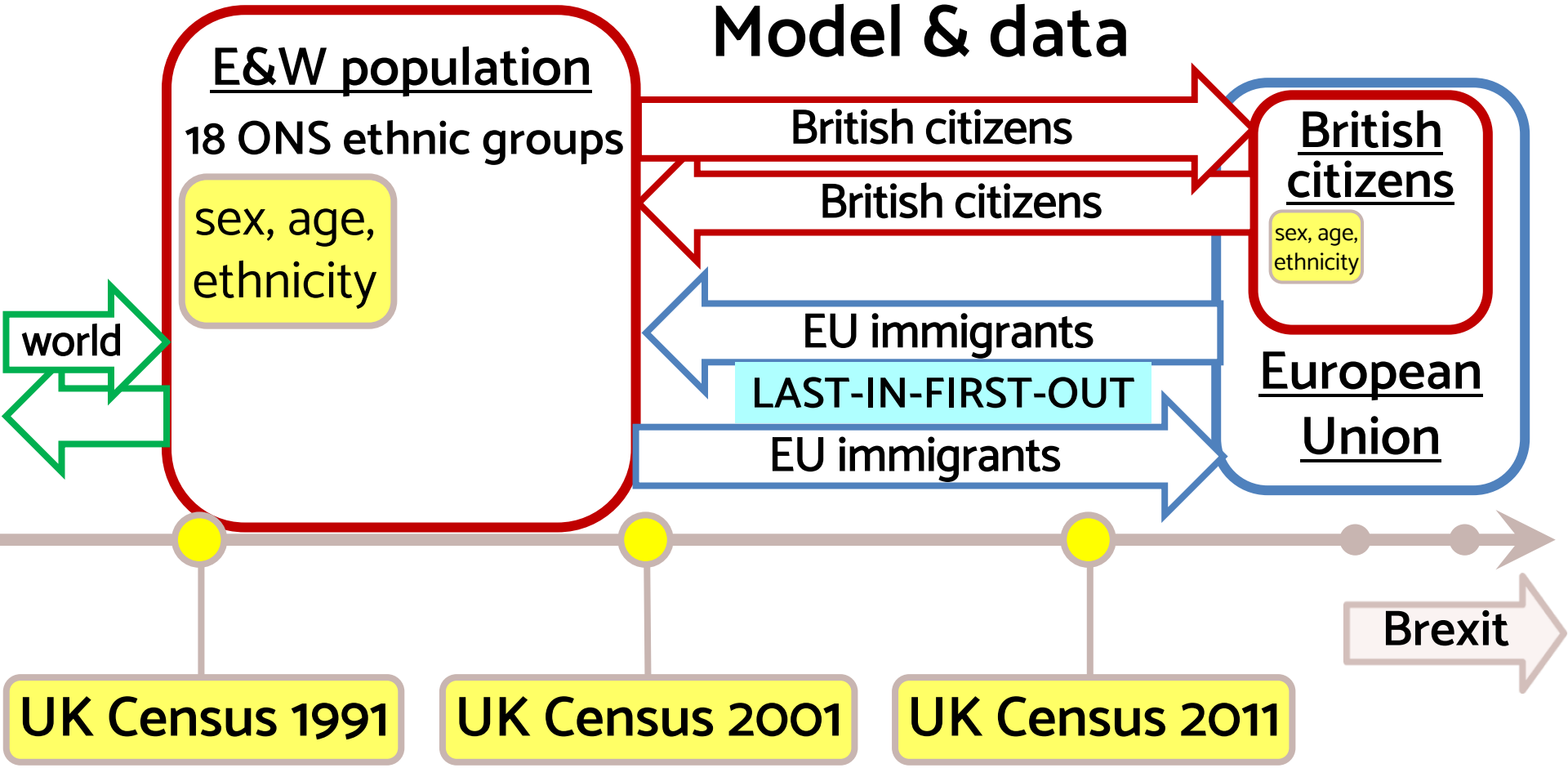
Pension-cost dependency ratio

number of people at or above the current retirement age
to people between ages 15 and that retirement age

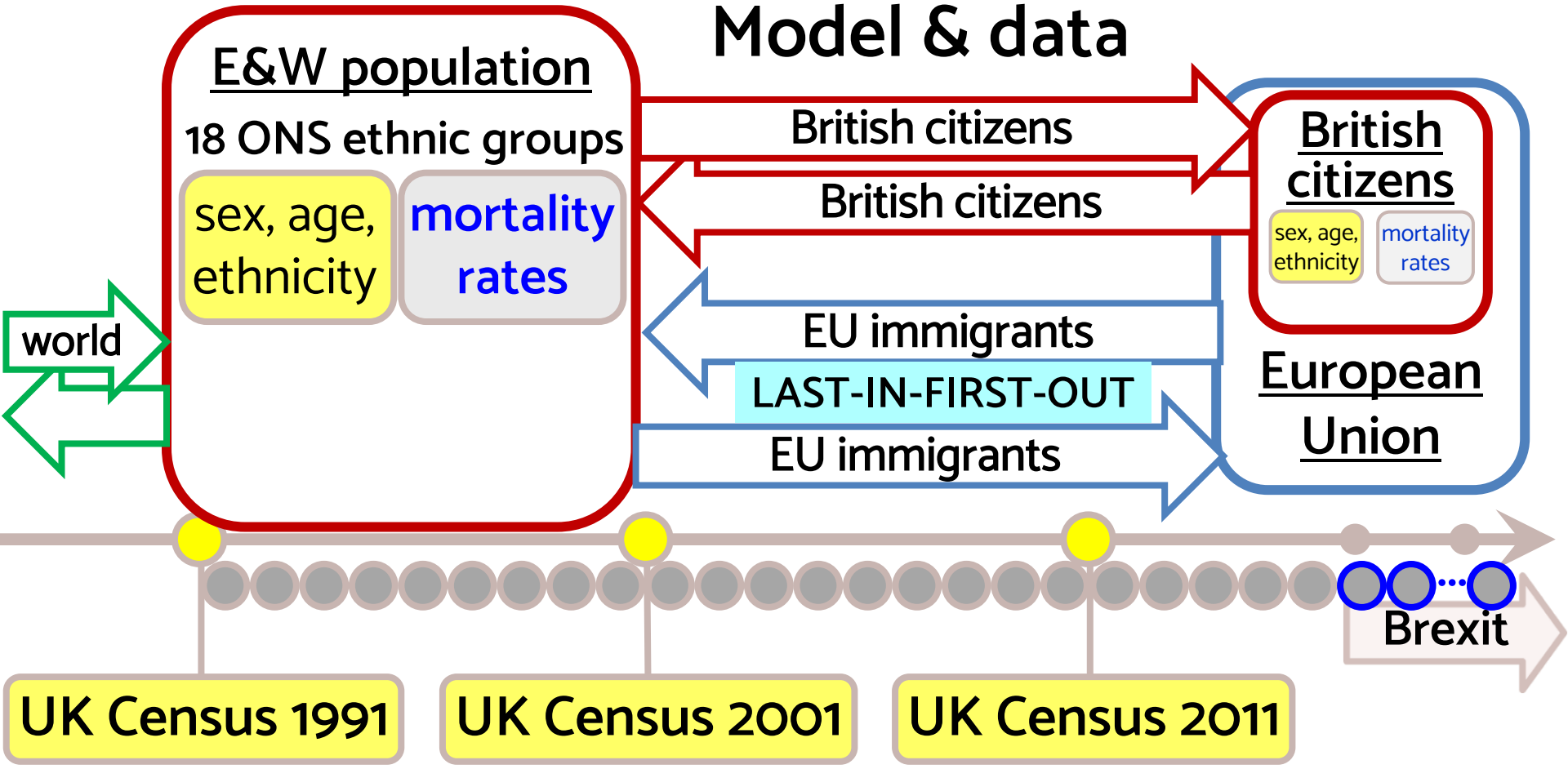
Model & data


E&W population
world
British citizens
British citizens
British citizens
EU immigrants
LAST-IN-FIRST-OUT
EU immigrants
European Union
Brexit

Model & data

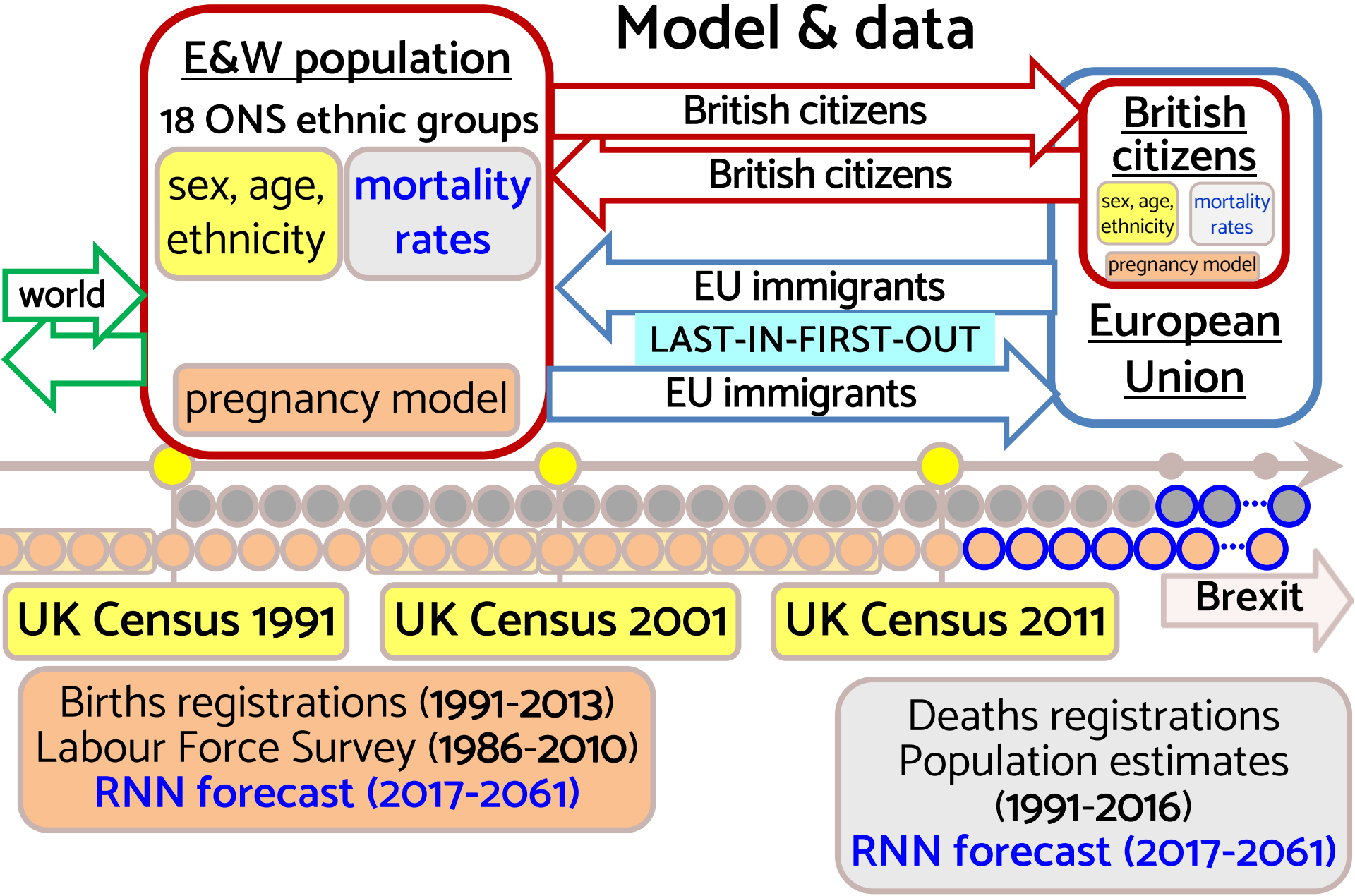

E&W population
18 ONS ethnic groups
sex, age, ethnicity
world
British citizens
British citizens
EU immigrants
LAST-IN-FIRST-OUT
EU immigrants
British citizens
sex, age, ethnicity
European Union
Brexit
UK Census 1991
UK Census 2001
UK Census 2011

Model & data



Deaths registrations
Population estimates
(1991-2016)
RNN forecast (2017-2061)

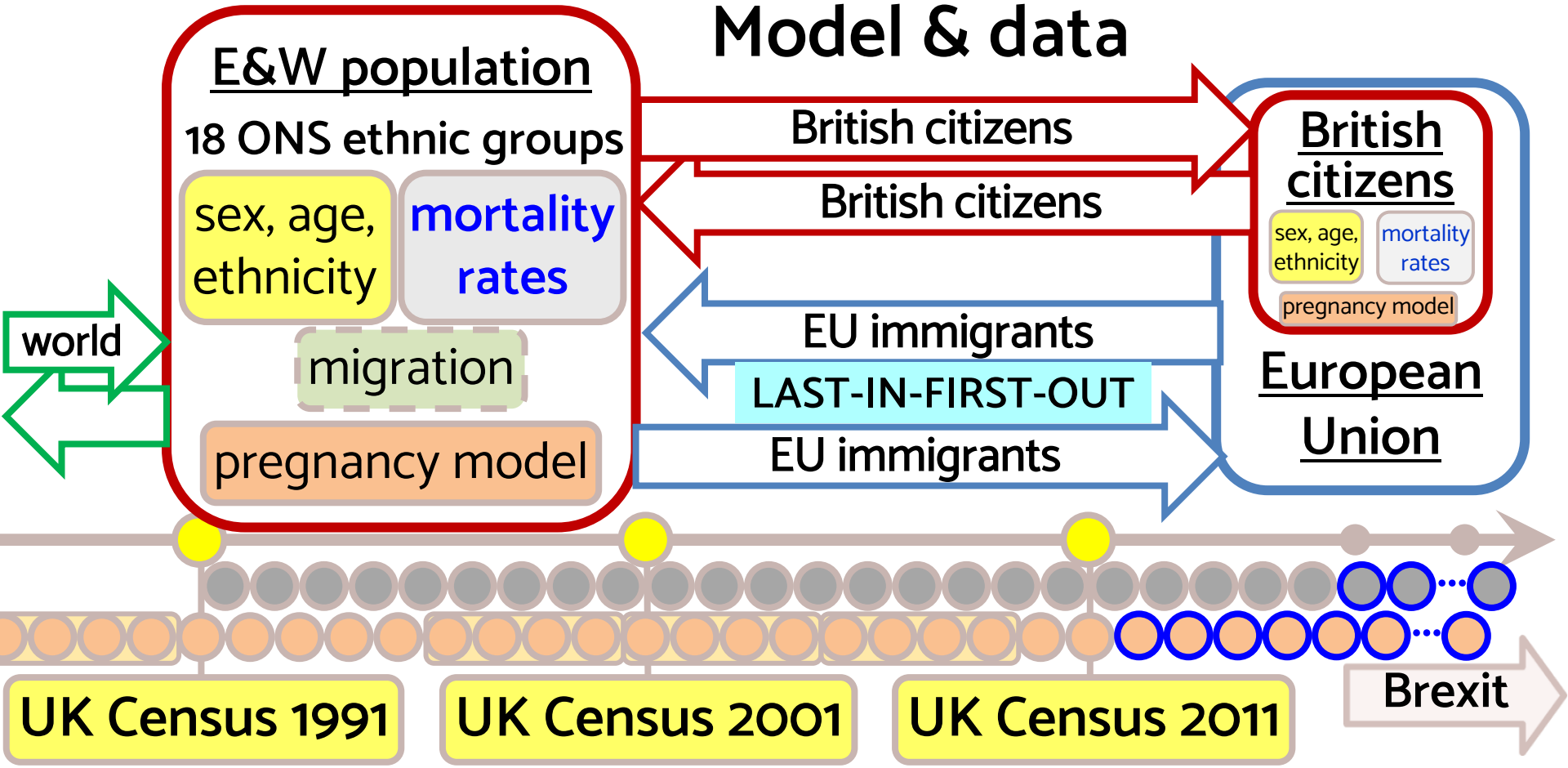
Model & data



Office for National Statistics

Coleman & Dubuc, Population Studies, 64(1), 19-41 (2010)
Simpson et al., Components of population change, 2008

Model & data



Births registrations (1991-2013)
Labour Force Survey (1986-2010)
RNN forecast (2017-2061)

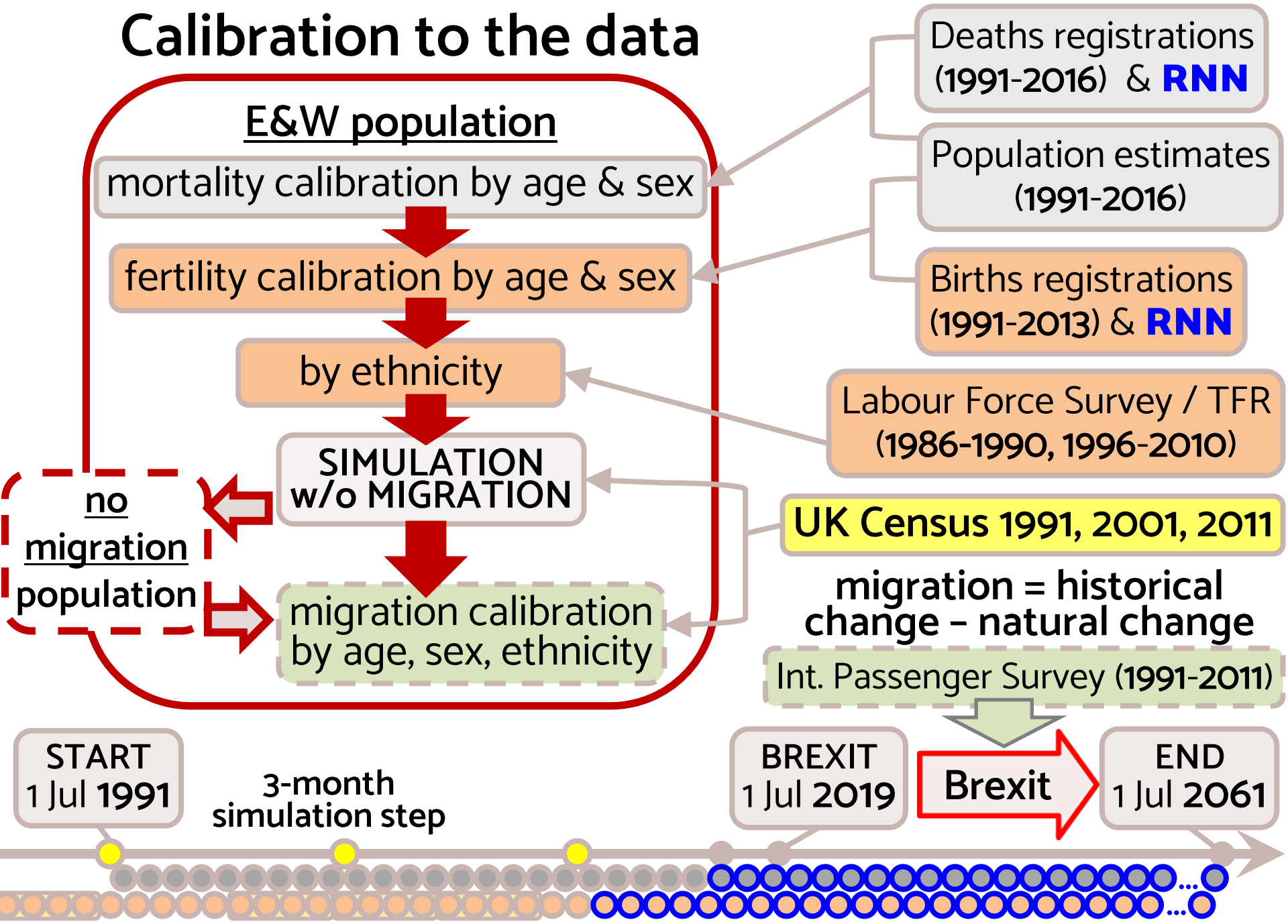
International Passenger Survey (1991-2011)
(by nationality)

Deaths registrations
Population estimates
(1991-2016)
RNN forecast (2017-2061)

Office for National Statistics

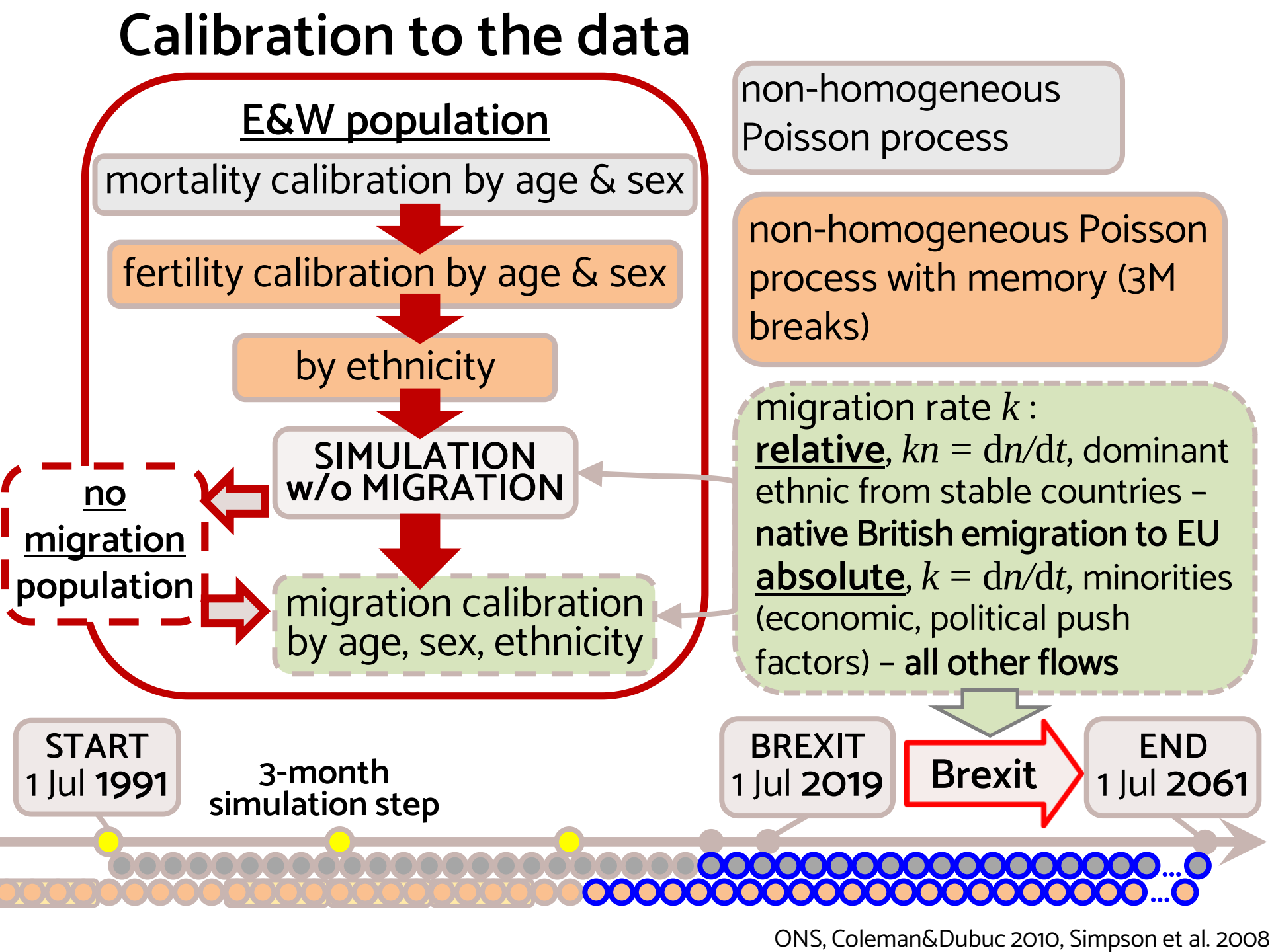
"Microsimulations of demographic changes in E&W under different EU referendum scenarios", *International Journal of Microsimulation*, 10(2), 2017

Calibration to the data



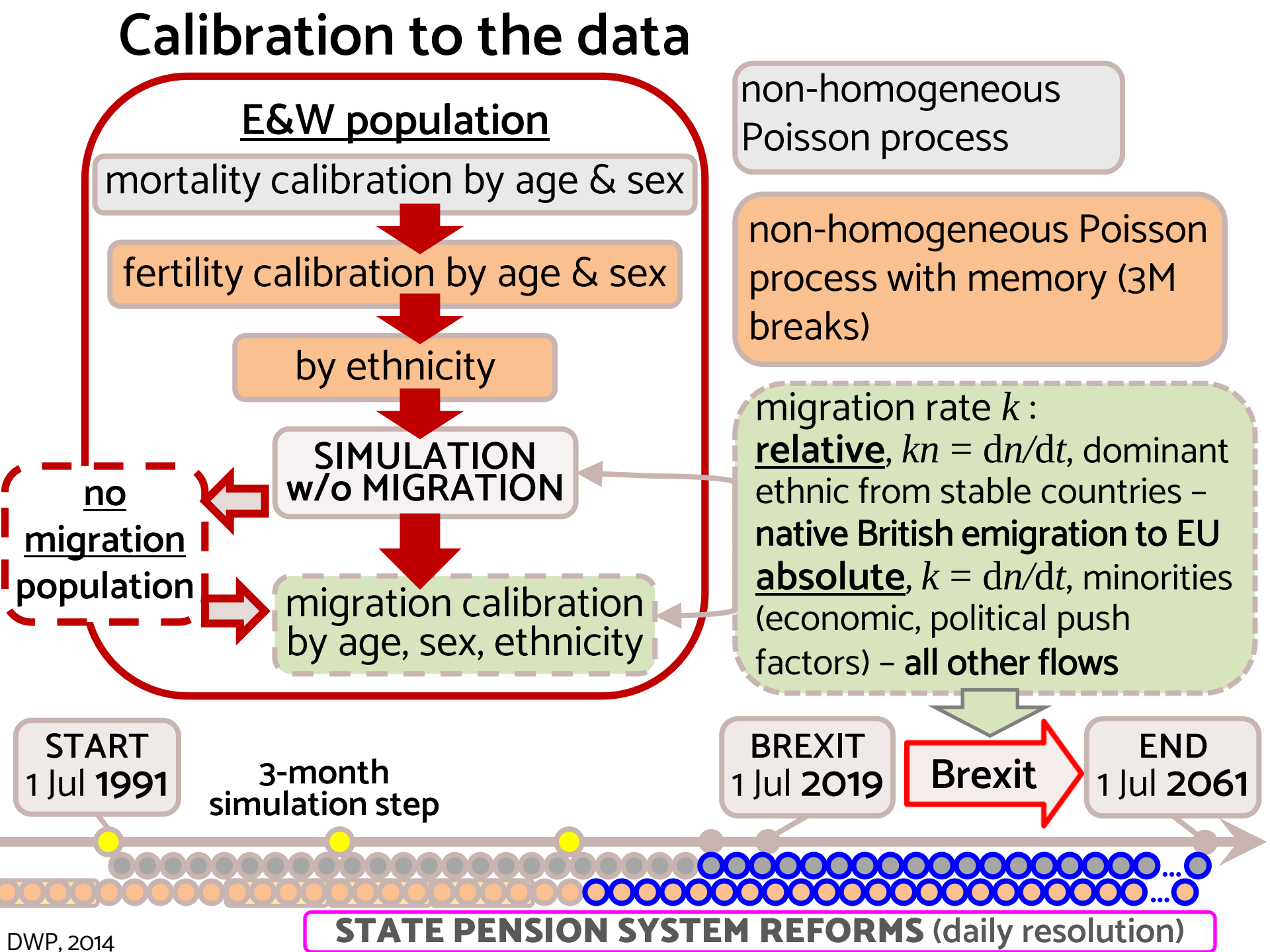
ONS, Coleman&Dubuc 2010, Simpson et al. 2008

Calibration to the data



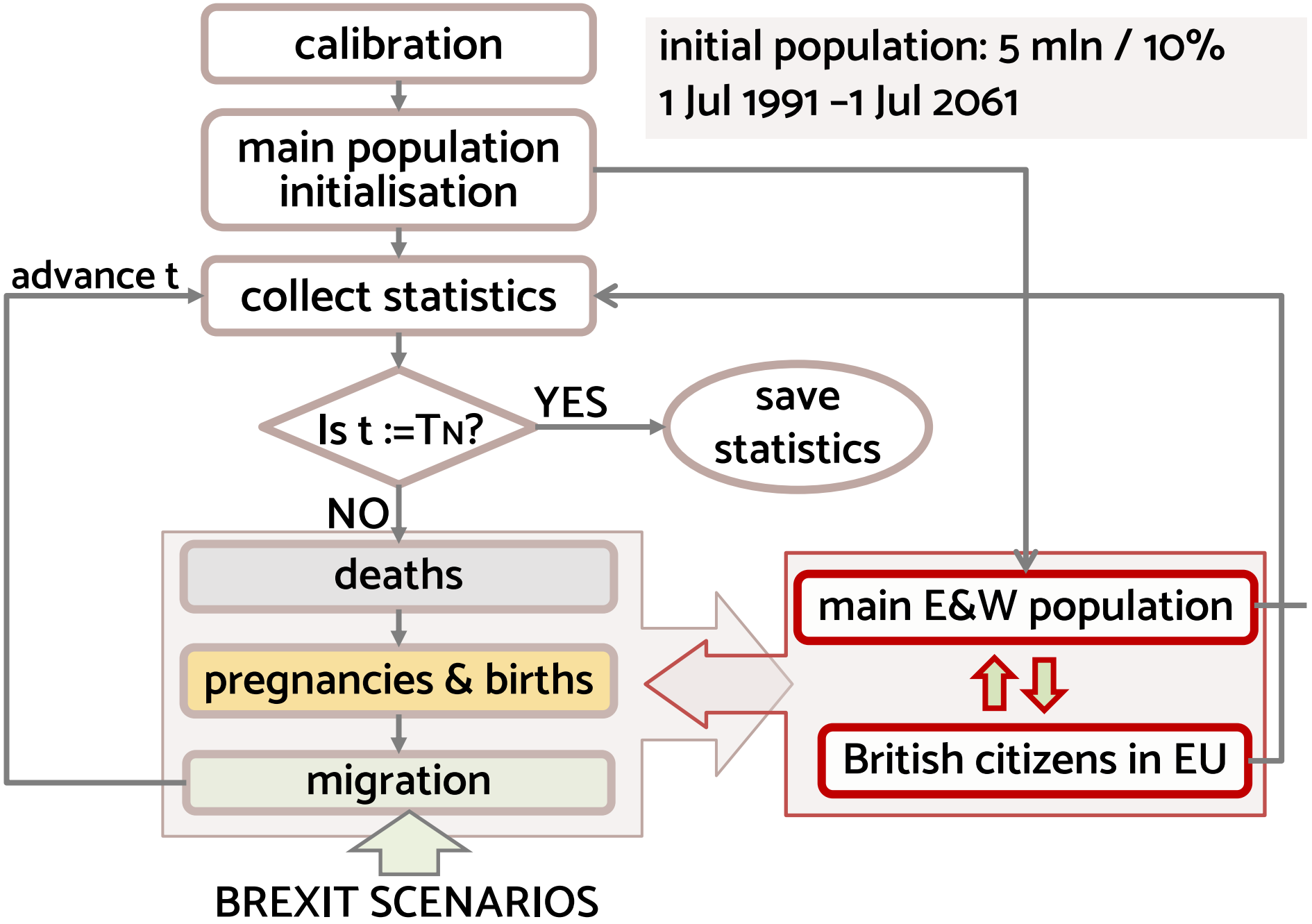
ONS, Coleman&Dubuc 2010, Simpson et al. 2008

Calibration to the data



DWP, 2014

Simulation


calibration
initial population: 5 mln / 10%
1 Jul 1991 –1 Jul 2061
main population initialisation
advance t
collect statistics
Is t :=TN?
YES
save statistics
NO
deaths
pregnancies & births
migration
main E&W population
British citizens in EU
BREXIT SCENARIOS

Post-Brexit scenarios: International migration changes

"Microsimulations of demographic changes in England and Wales under different EU referendum scenarios", *International Journal of Microsimulation*, 10(2), 2017

exodus of EU immigrants from the UK to EU

wave of returns of the British from the EU

changing migration of British citizens to/from EU

changing migration of EU immigrants to/from the UK

Post-Brexit scenarios

10%

2019-2021

10%

"Soft"

80%

avg 2000&2011

Post-Brexit scenarios

10%

2019-2021

10%

"Soft"

80%

avg 2000&2011

70%

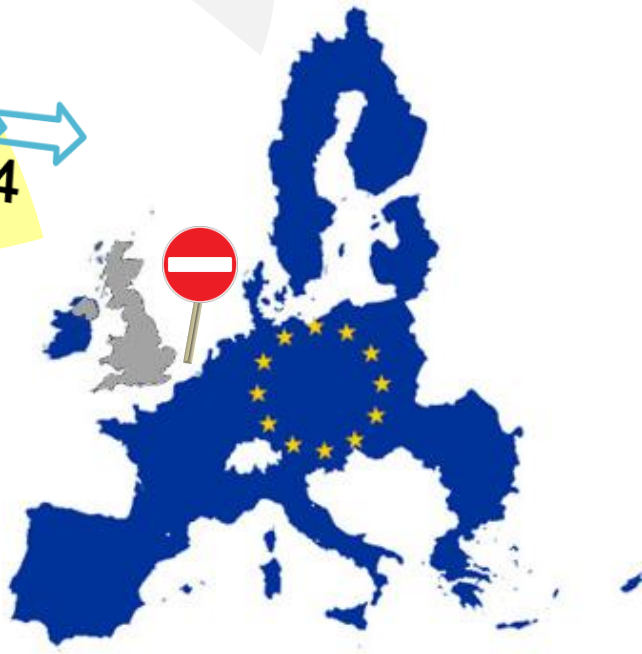
2019-2021

80%

"Hard"

30%

pre-2004



Post-Brexit scenarios

10%

2019-2021

10%

"Soft"

80%

avg 2000&2011

70%

2019-2021

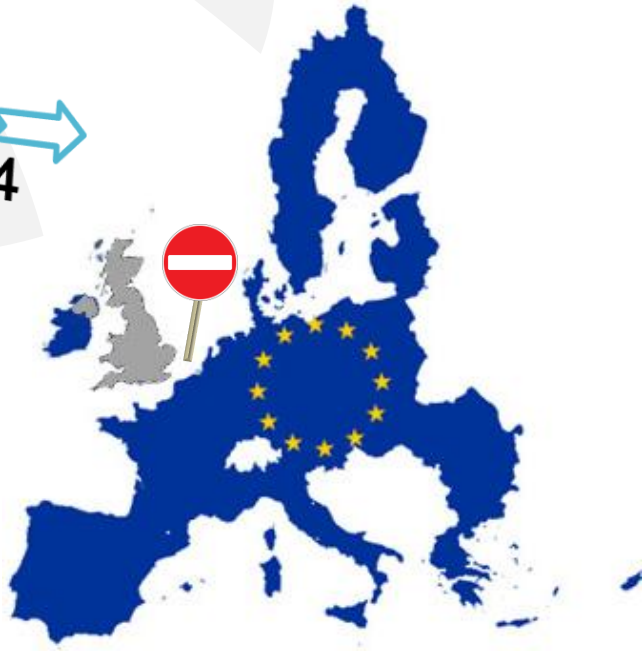
80%

"Hard"

30%

pre-2004

"Bremain"



Post-Brexit scenarios

10%

2019-2021

10%

"Soft"

80%

avg 2000&2011

70%

2019-2021

80%

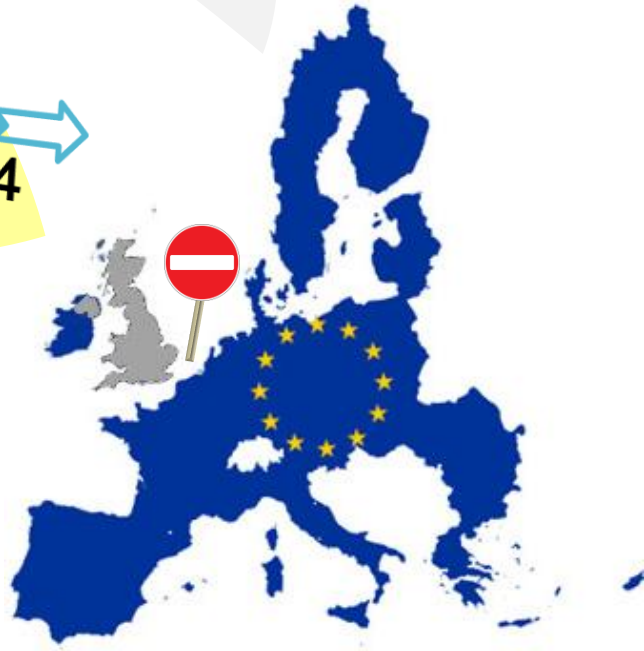
"Hard"

30%

pre-2004

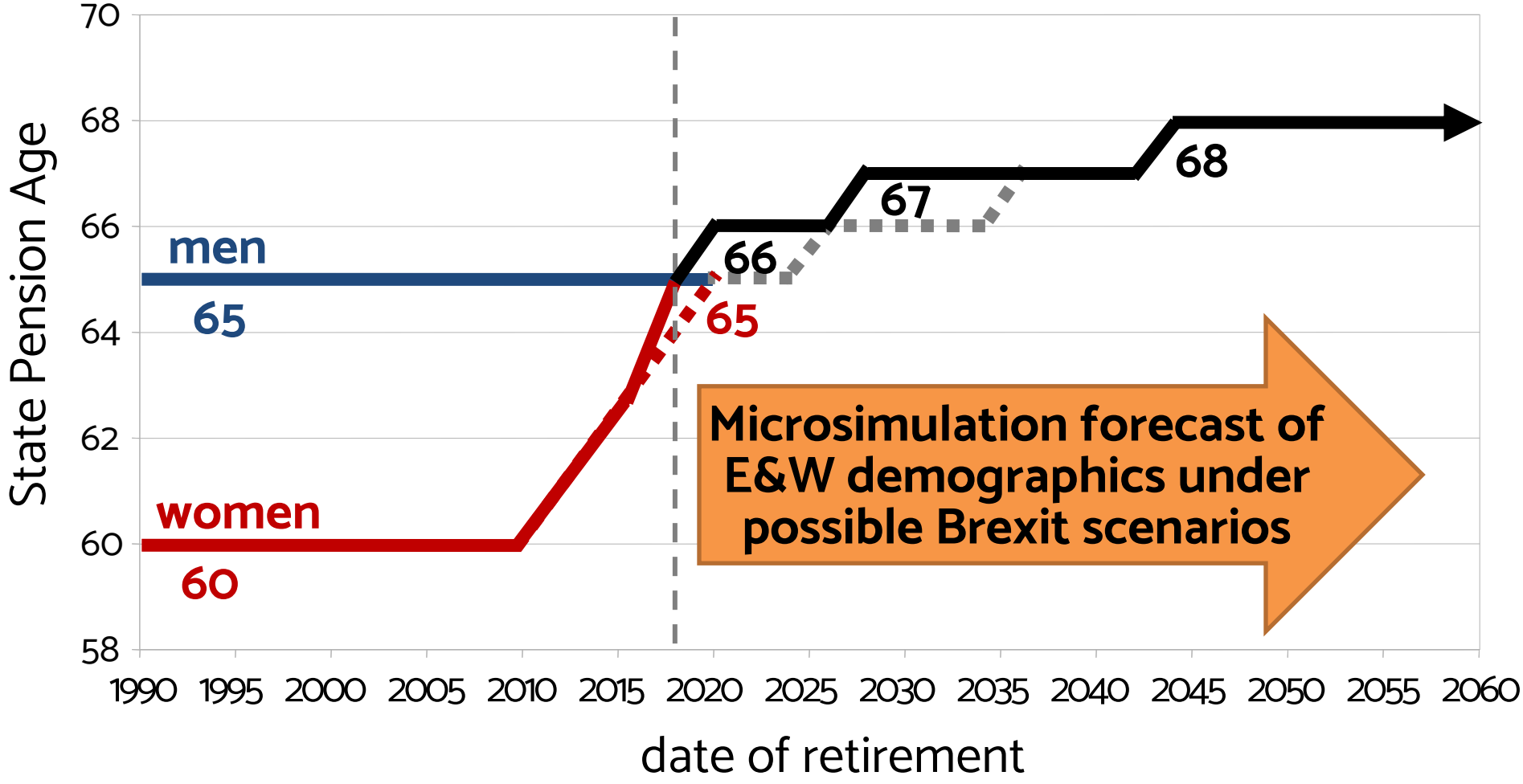
x1.3

"Bremain"



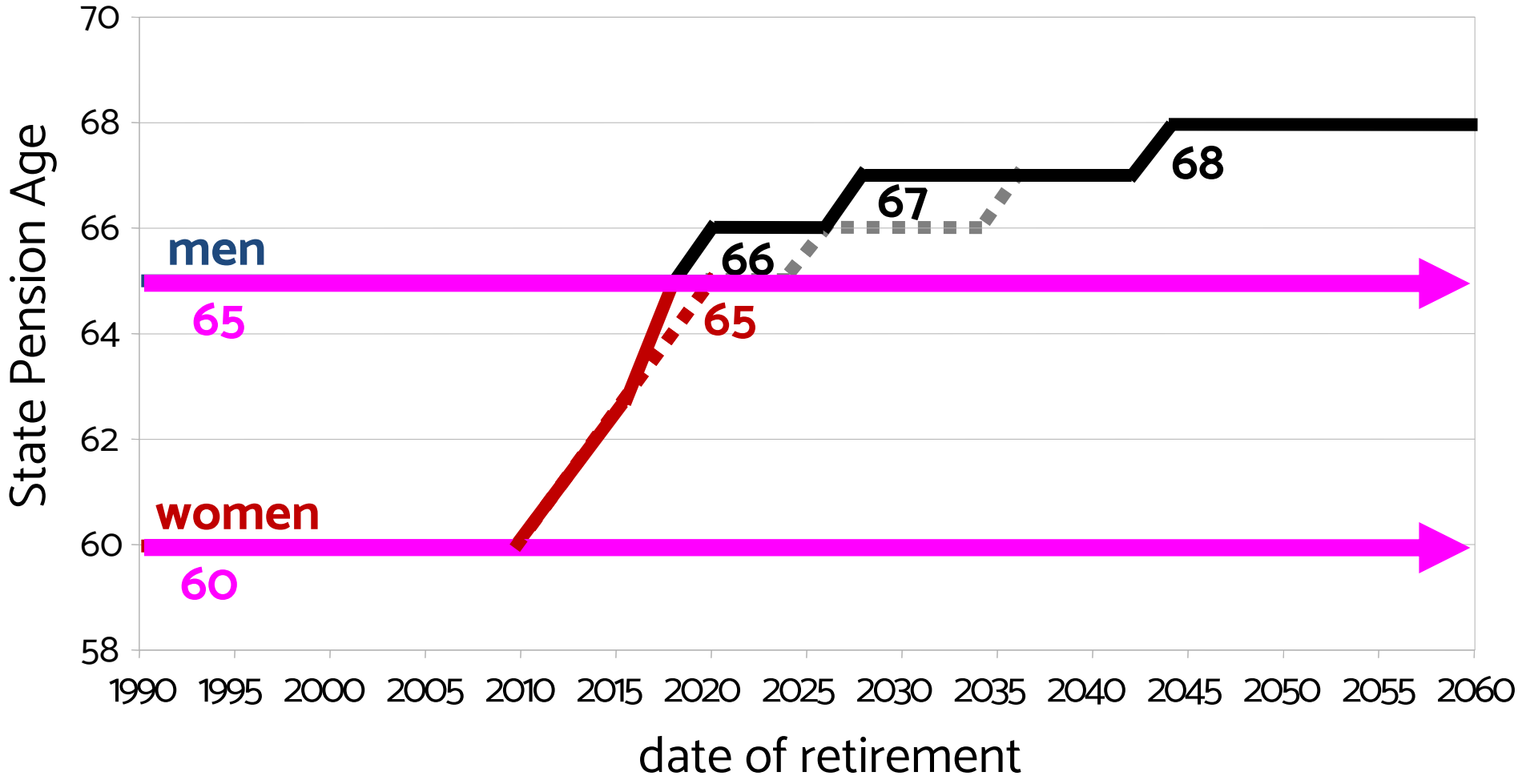
"Hello World!"

Forecasting the impact of state pension reforms in post-Brexit E&W



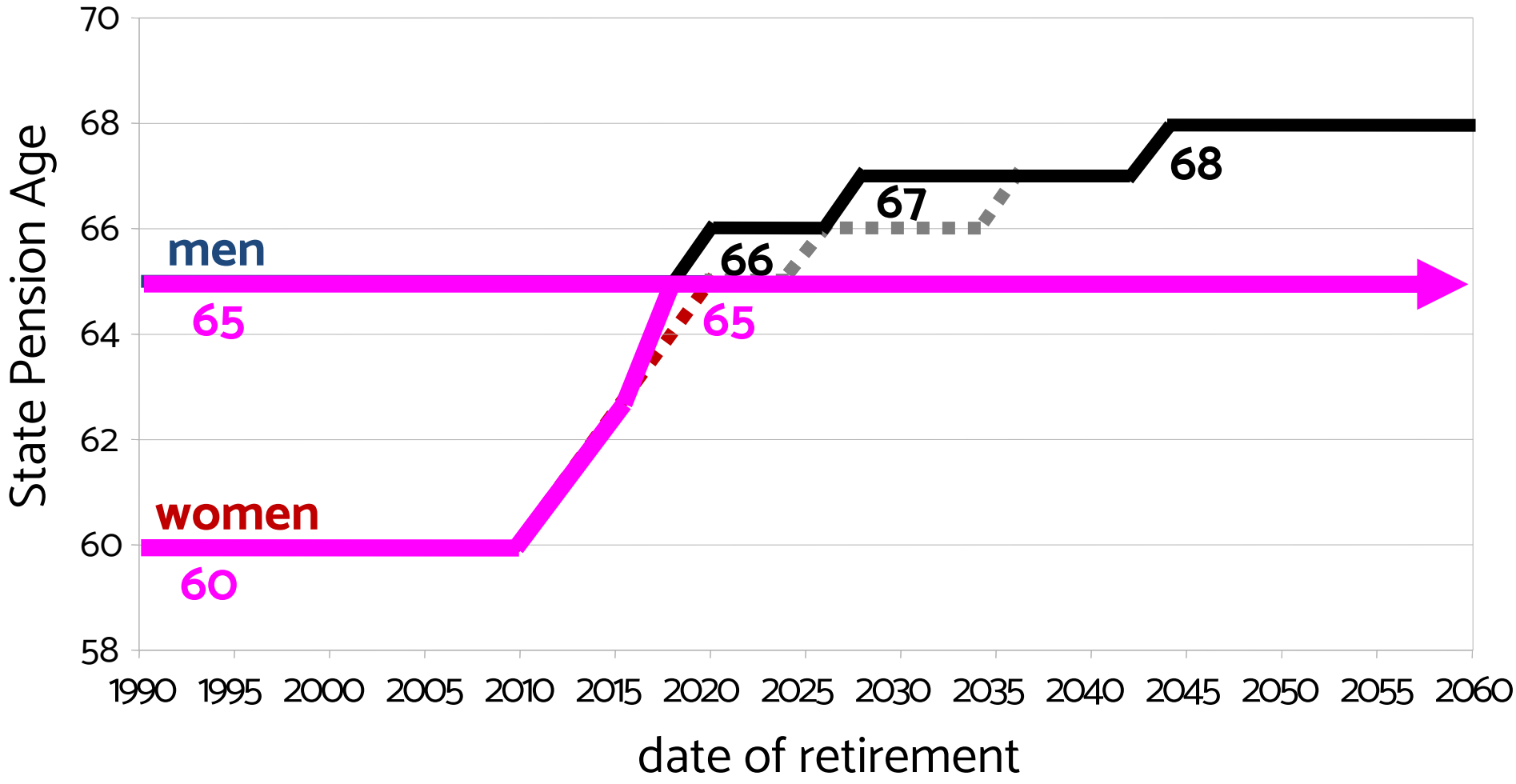
State pension system scenarios:

1. men 65 / women 60



State pension system scenarios:

2. Equal SPA 65



State pension system scenarios:

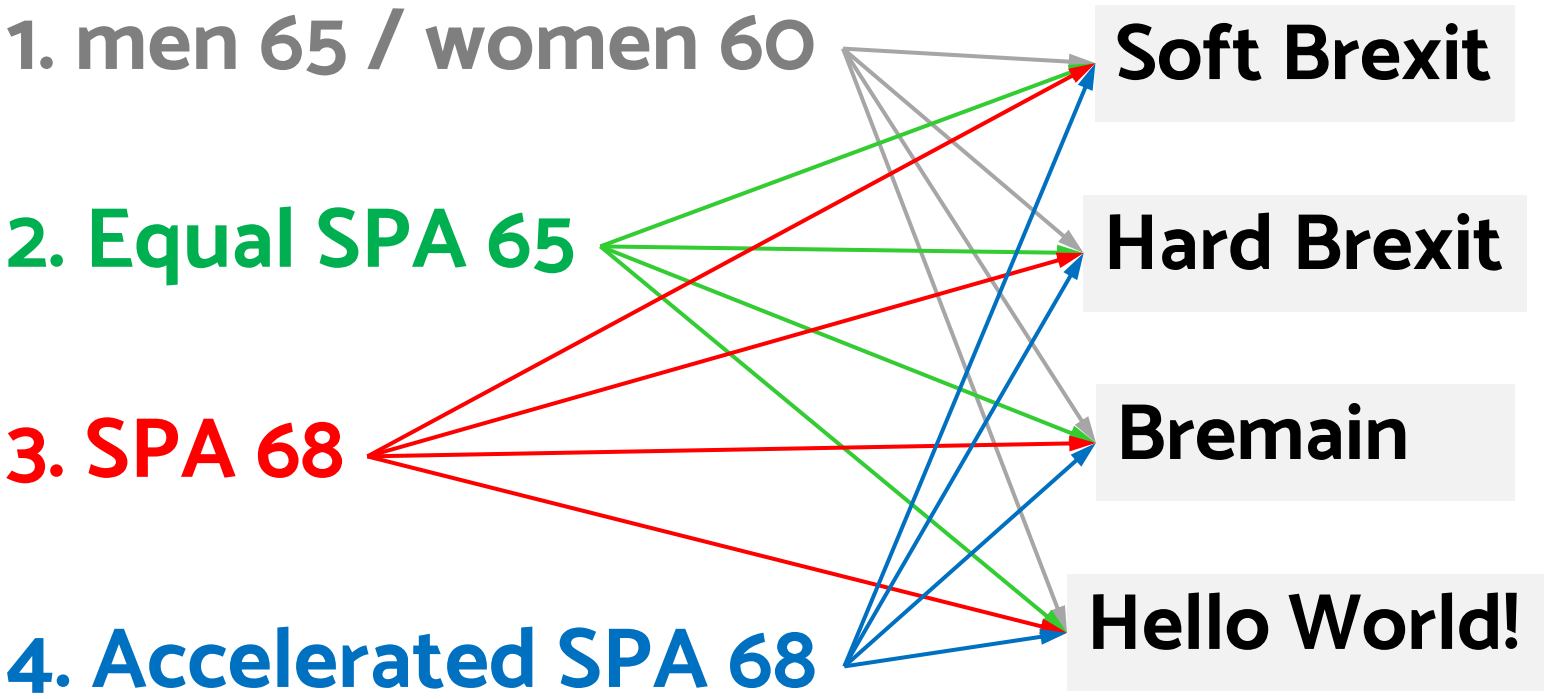
3. SPA 68

State pension system scenarios:

4. Accelerated SPA 68

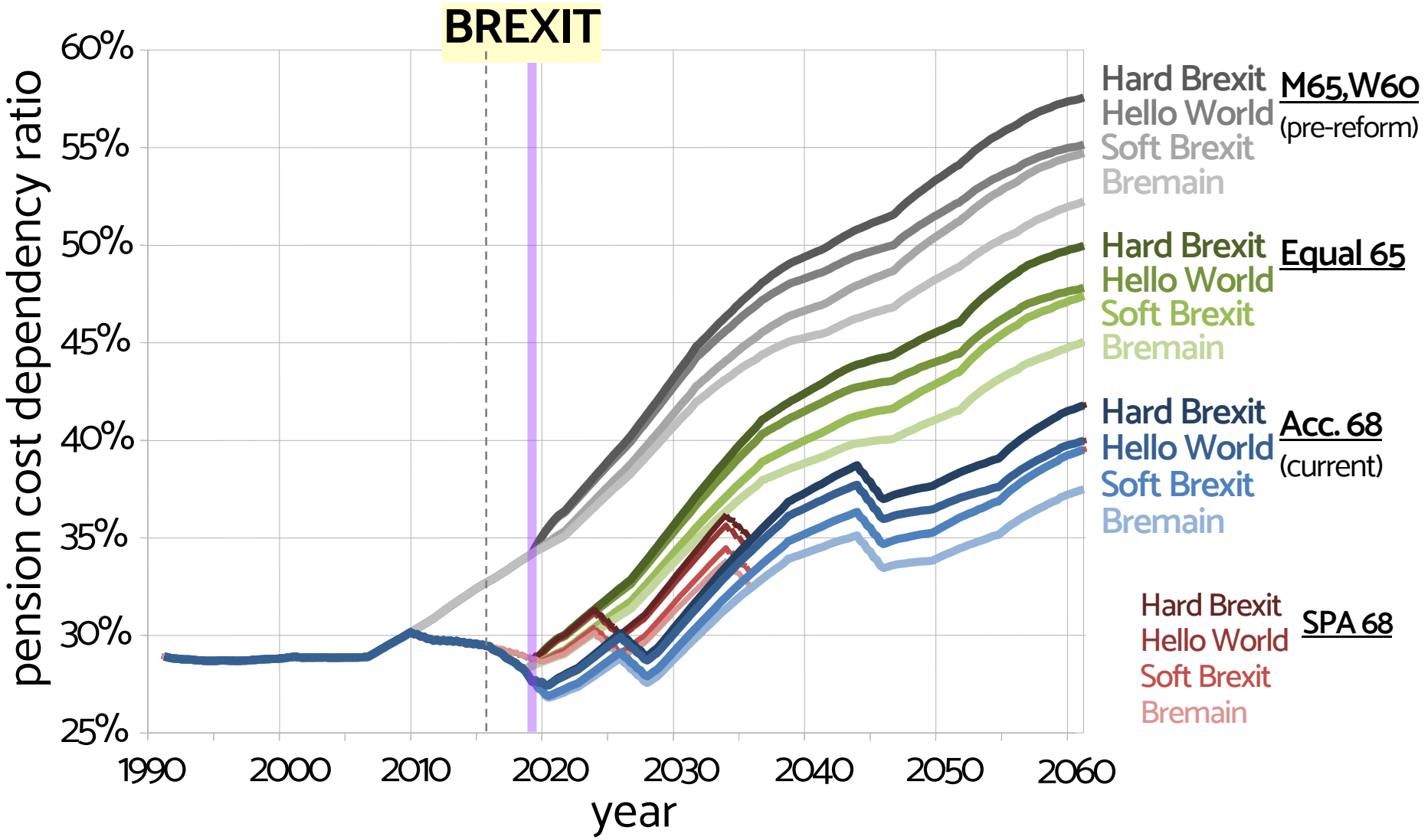
State pension system scenarios: Pension-cost dependency ratio

number of people at or above the current retirement age to people between ages 15 and that retirement age



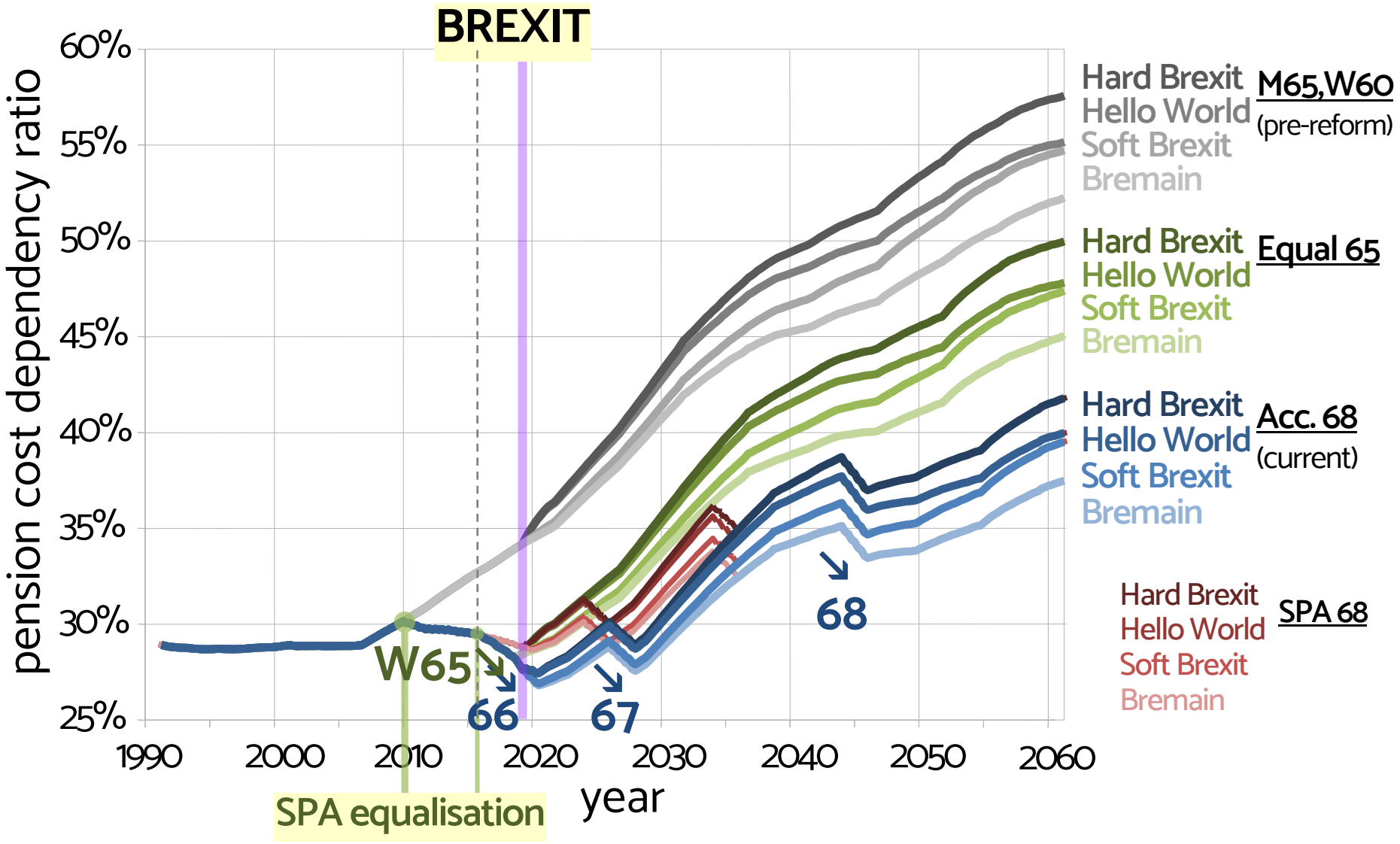
Pension-cost dependency ratio

number of people at or above the current retirement age
to people between ages 15 and that retirement age



Pension-cost dependency ratio

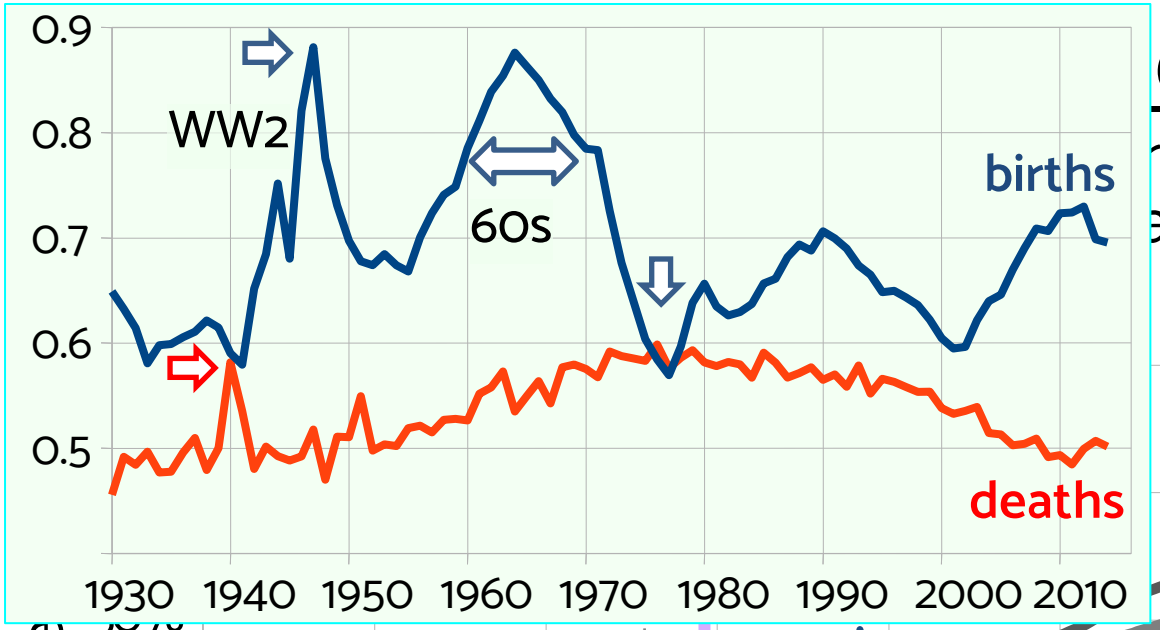
number of people at or above the current retirement age
to people between ages 15 and that retirement age

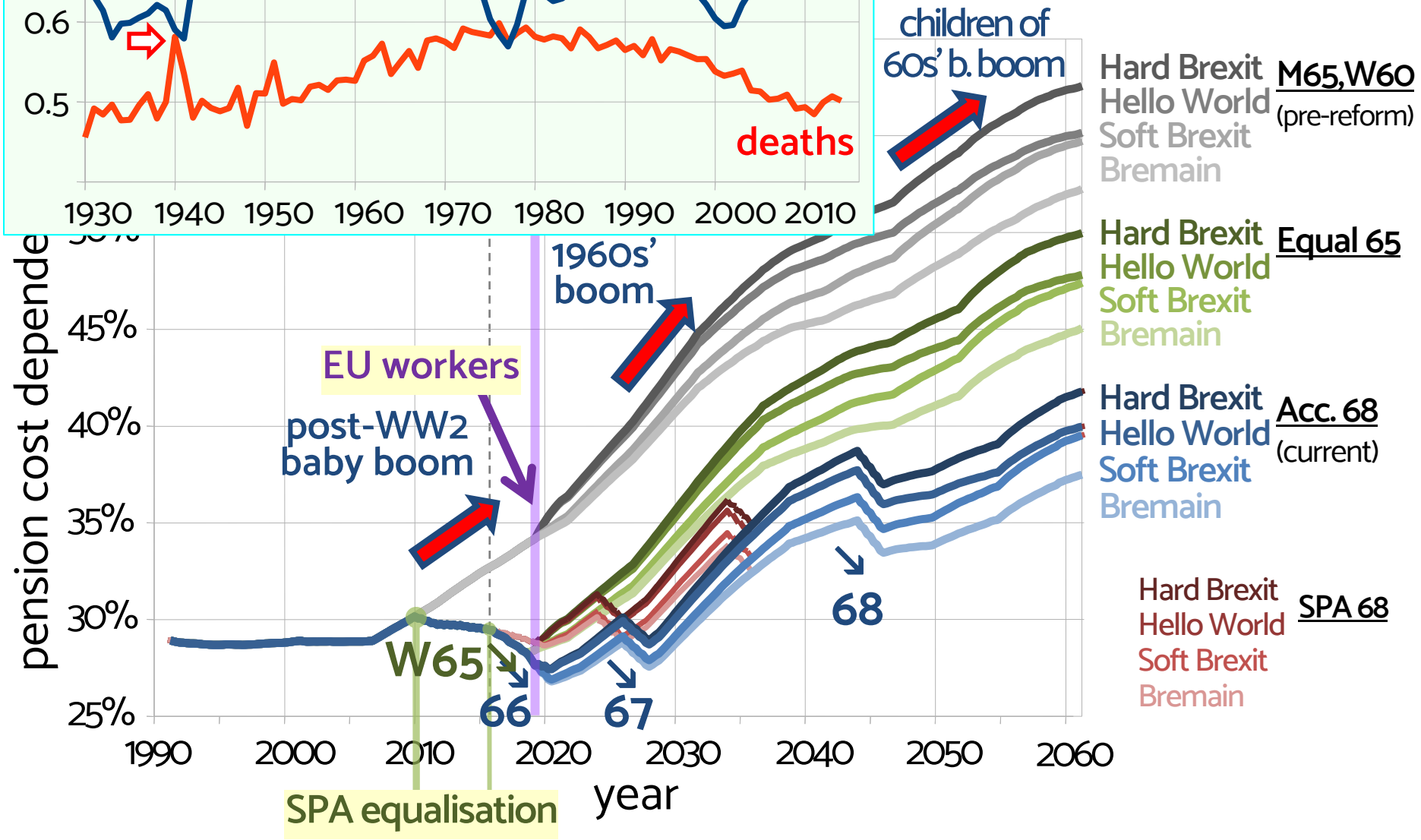


ency ratio

rrent retirement age

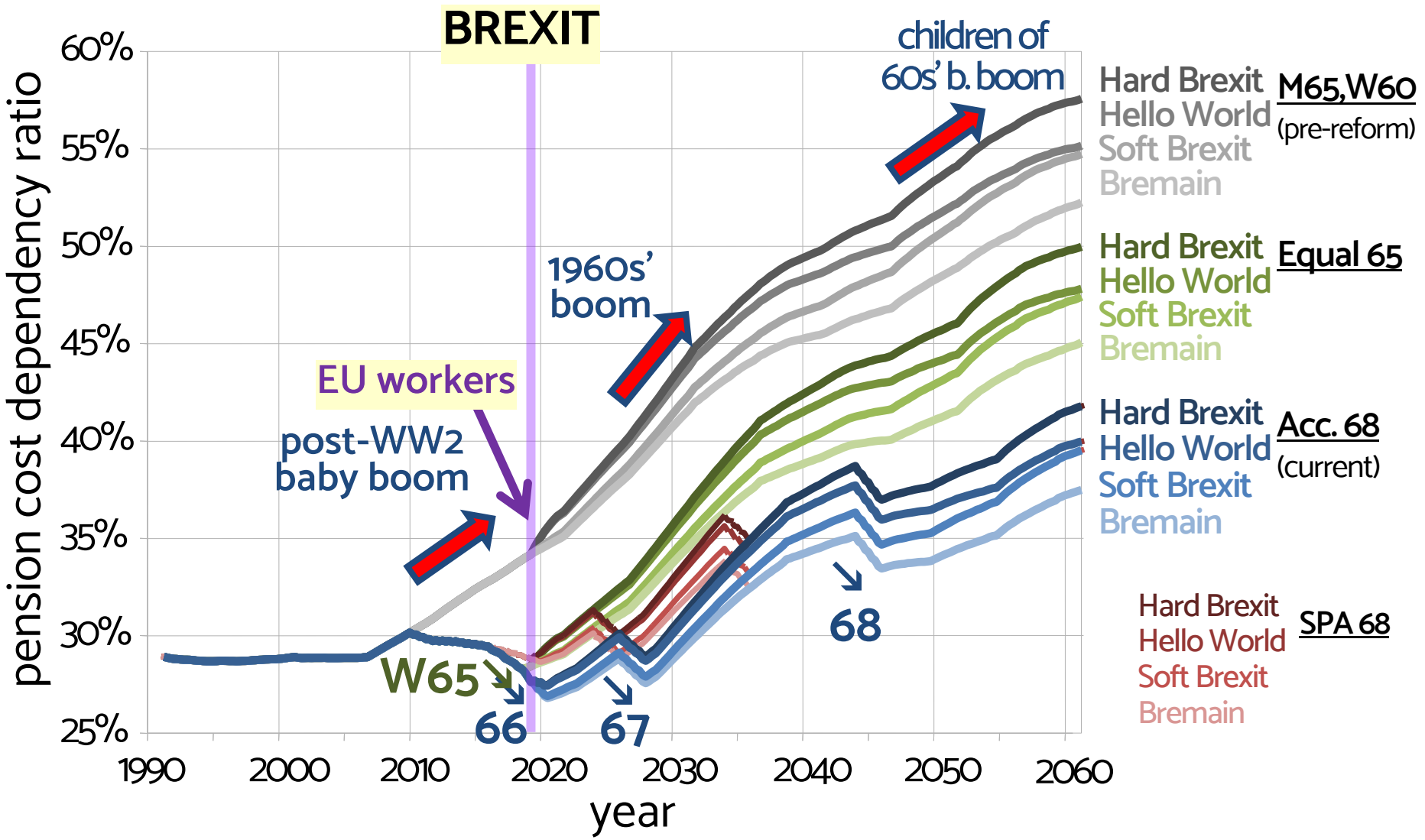
at retirement age





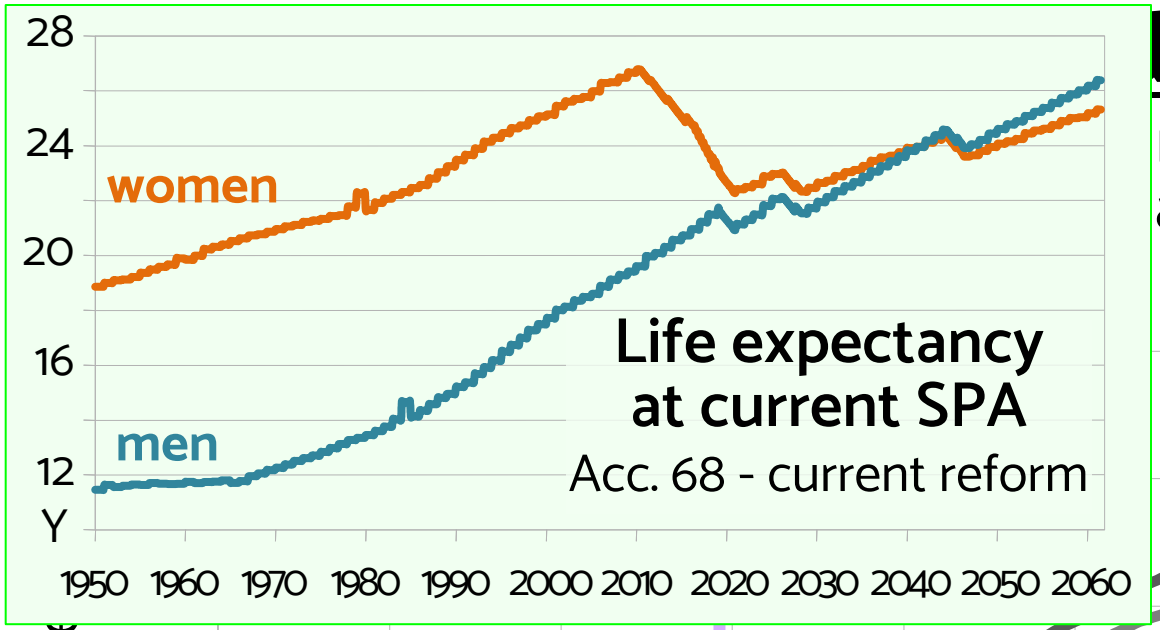
Pension-cost dependency ratio

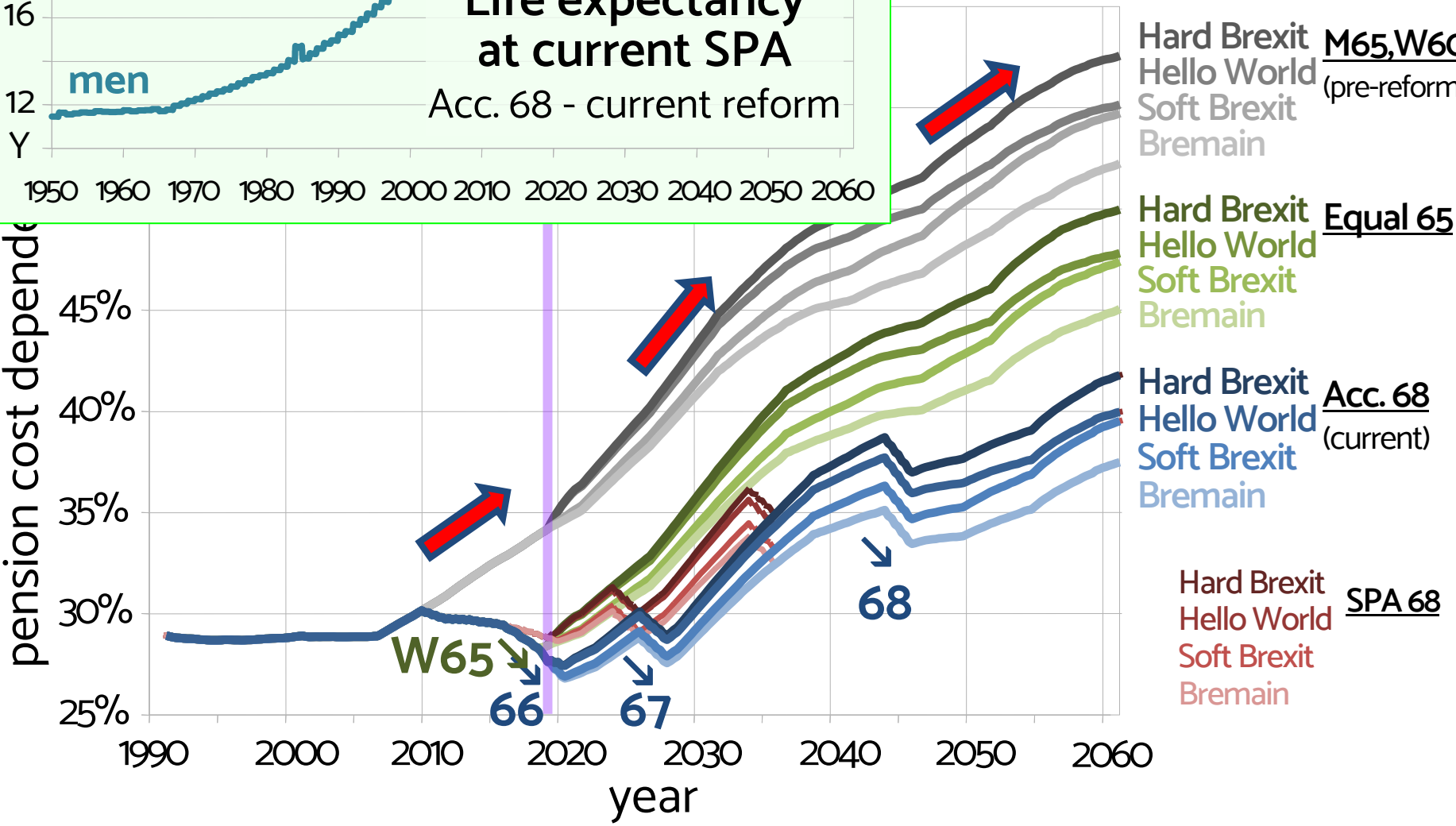
number of people at or above the current retirement age
to people between ages 15 and that retirement age



lency ratio

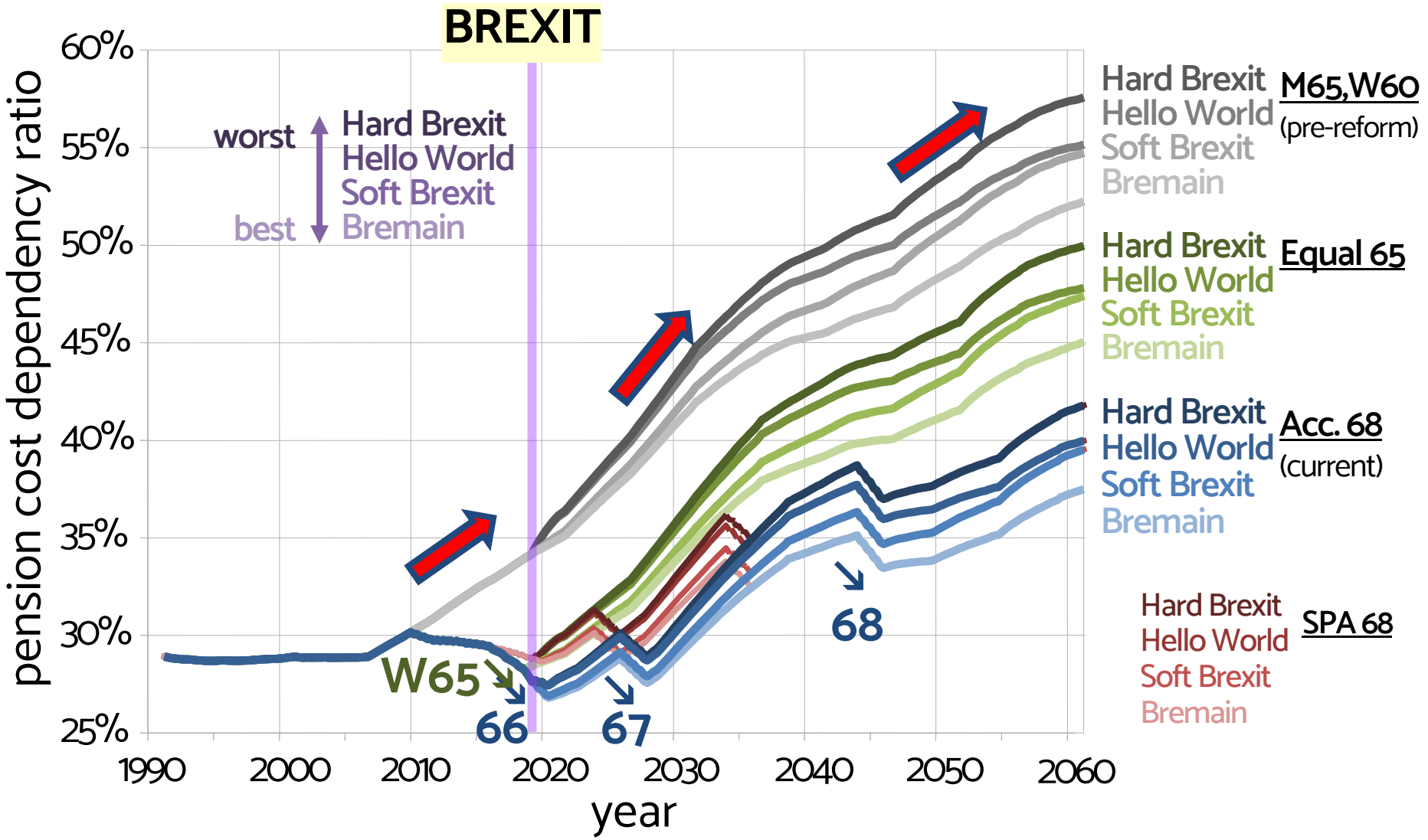
rrent retirement age
at retirement age





Pension-cost dependency ratio

number of people at or above the current retirement age
to people between ages 15 and that retirement age



Summary

- Microsimulation forecast shows that SPA reforms in England &Wales stave off the "pension crisis" due to population ageing for the next decade
- The reforms offer a generous and fair share of life in retirement (~26%)
- International migration changes after Brexit will affect the pension cost dependency ratio – secondary effect to population ageing
- Deep learning models of mortality & fertility rates discover their complex trends (survival curve rectangularisation with stronger age of death compression for women, regression to the mean, men outliving women in the future)
- Neural networks "think" that gender inequality shrinks women's chances of living till a very old age?

Thank you for your attention!

&

for wonderful and inspiring time in Tokyo and Łódź

Supplemental materials

Agnieszka Werpachowska

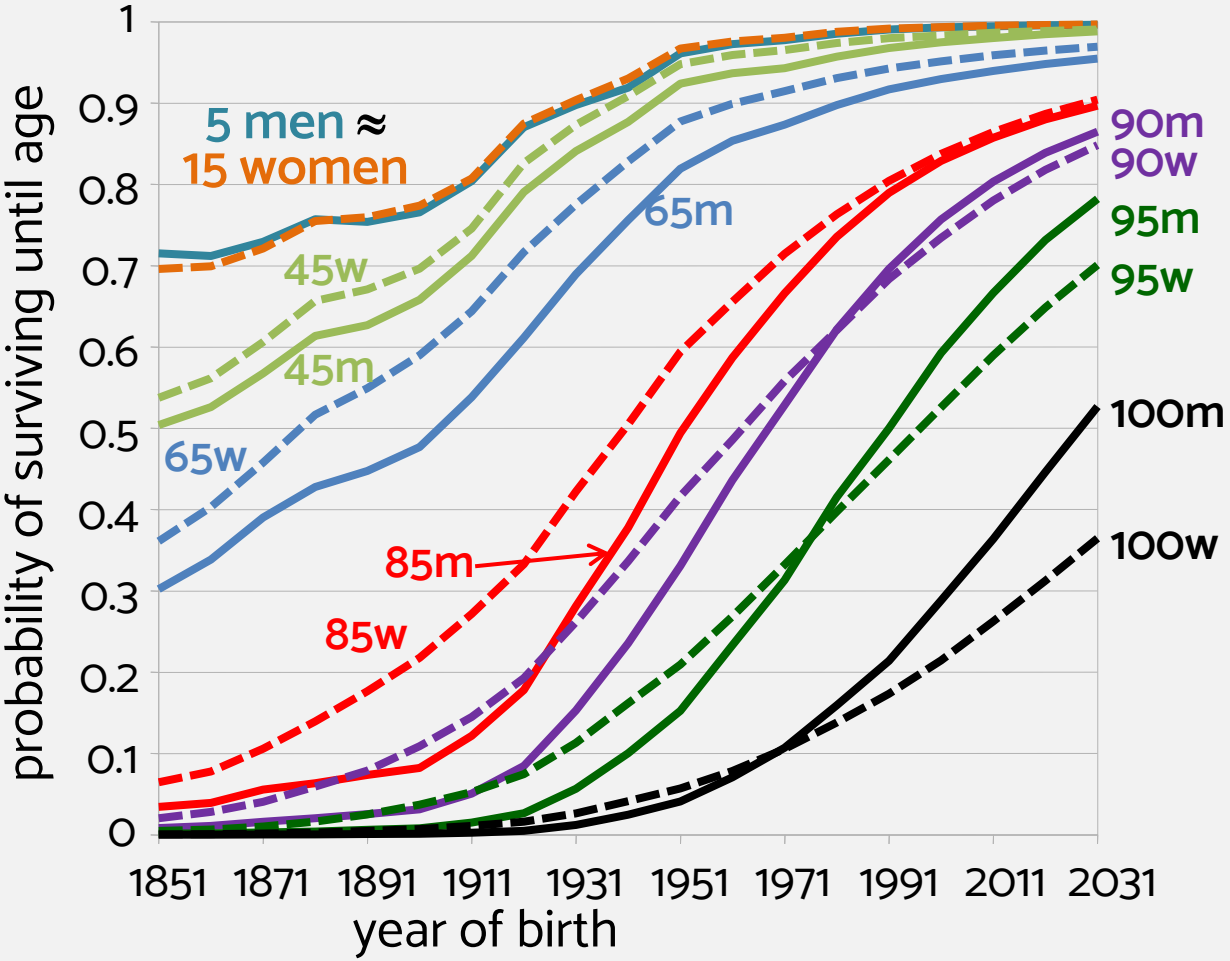
AVERISERA

MATHEMATICAL INVESTIGATIONS OF REALITY

averisera.uk

How men's lifespan outpaces women's?

Theories why women live longer: extra X chromosome, less testosterone, "female jogger heart", height (more body cells / more harmful mutations); research on mice



❑ONS (2014): "There is a long-running trend since the 1970s for male life expectancy to catch up with female..."

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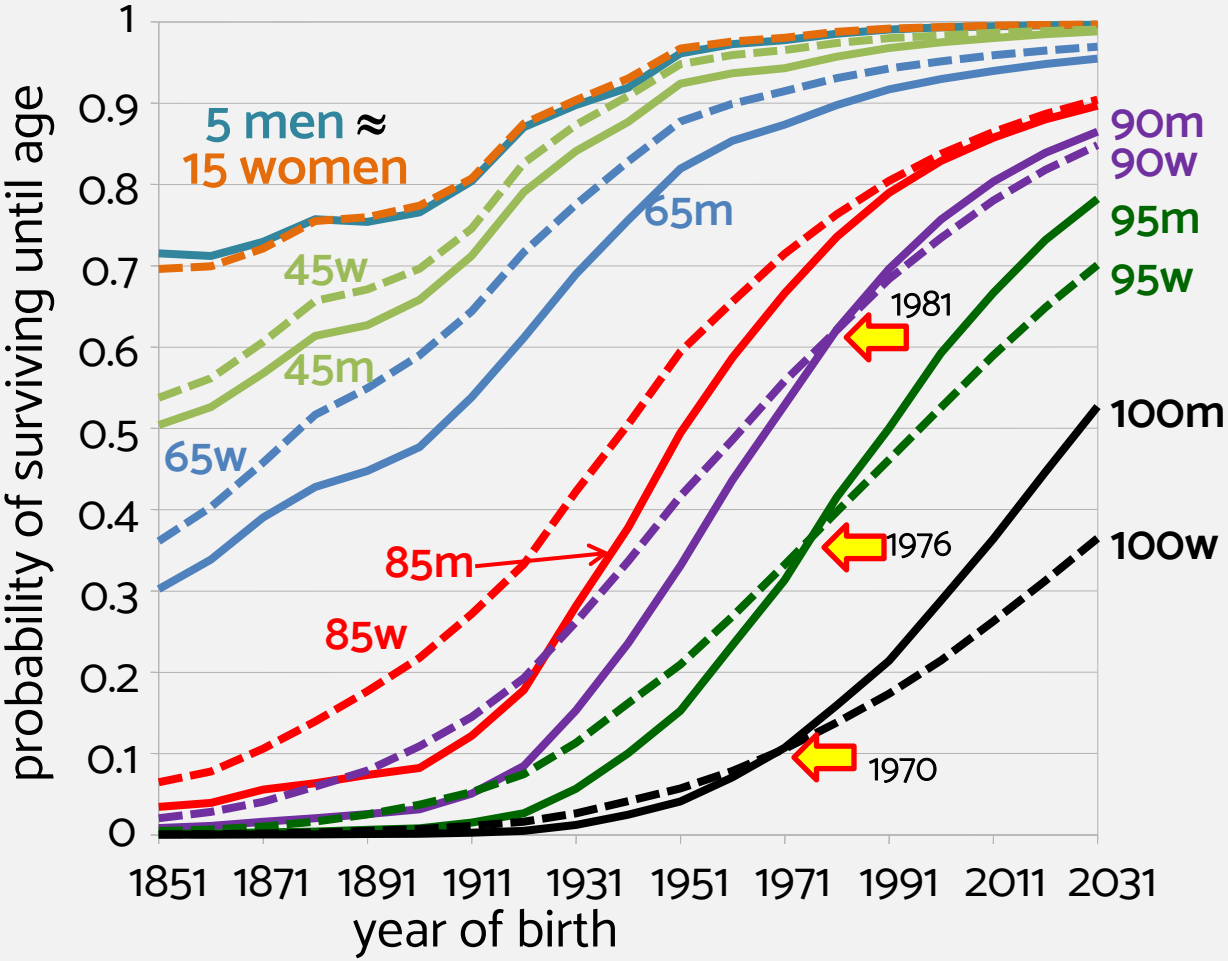

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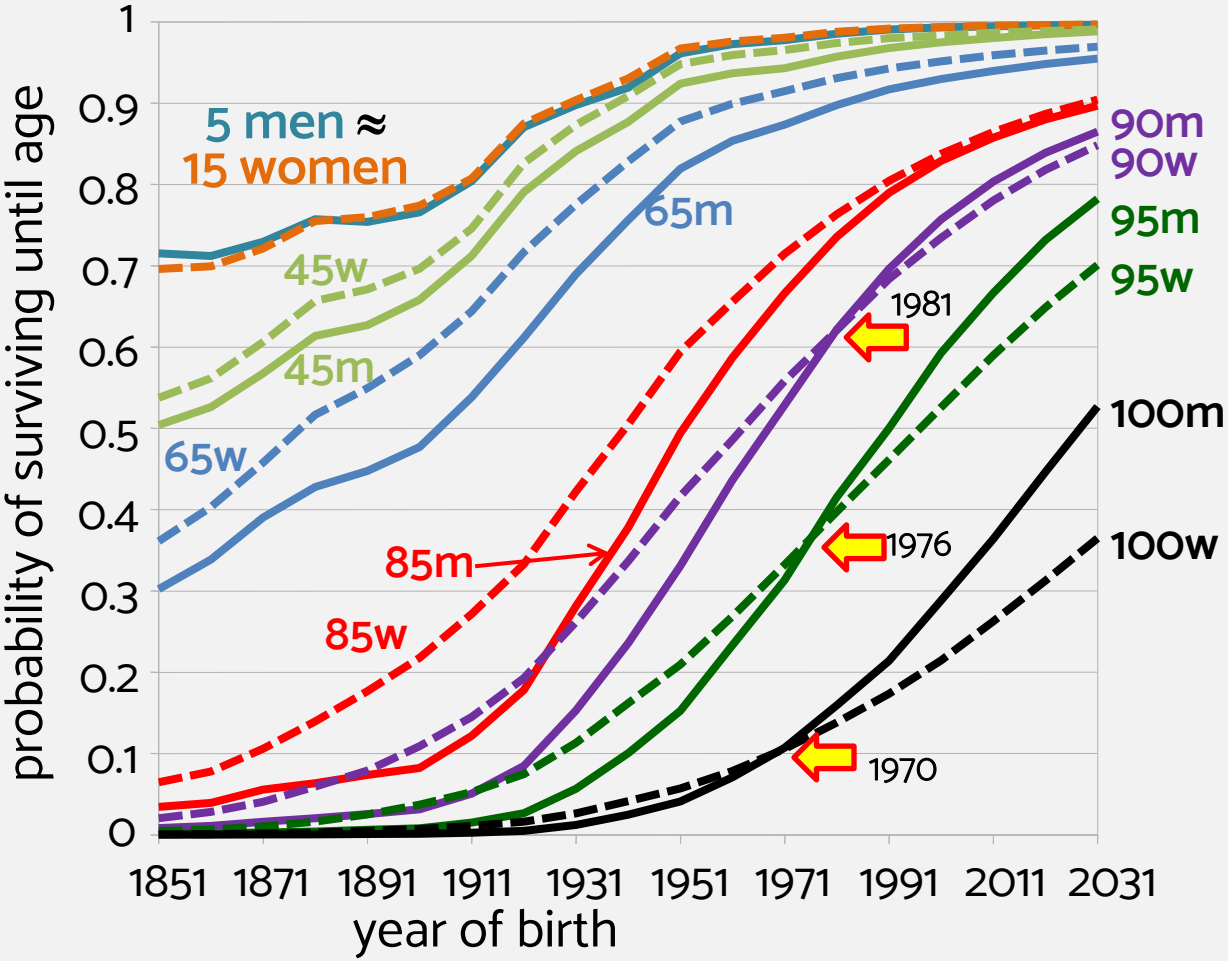

2070
era of men?

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